New Approaches for Debt Sustainability Analysis

Eduardo Cavallo
Research Department

Prepared for the V Workshop of the LAC Debt Management Specialists Group

October 19-20, 2009

Roadmap

- Motivation.
- Background: A dichotomy of DSA methodologies.
- Fan Charts for DSA.
- Current work on DSA at the IDB

Motivation

- Traditional DSA frameworks rely on medium term simulations of the debt-to-GDP ratio given specific macroeconomic forecasts and fiscal policy assumptions.
- A key issue with the standard DSA's is the omission of uncertainty, for example about income growth, interest rates, fiscal policies, and exchange rates.
- In principle assessing debt sustainability involves two things:
  - (1) contemplating the probable future debt paths given the existing projections about the fiscal policy stance and the expected evolution of the main macroeconomic variables.
  - (2) assessing the country's risk profile given its history and also the pattern of shocks that typically beset the country.
  - Most DSA approaches deal with either (1) or (2), but few attempts have been made to combine both.

Background: A dichotomy of DSA methodologies

- Endogenous debt dynamics (IMF): medium term projections of the debt dynamics with “stress” tests.
  - Forward looking, but no uncertainty.
- Risk Management Approach (Garcia and Rigobon, 2004) uses regression analysis with subsequent simulation.
  - Captures regularities prevalent in the data, but is entirely backward looking.
- Fan Charts for DSA (Arizala, Castro, Cavallo and Powell, 2008) argues that the RMA can be enhanced with the consideration of external projections and/or subjective measures.
  - For certain countries, simulations that postulate independent paths for the relevant variables may miss be point, but at the same time, you might want to consider possible *structural breaks* that are not easily captured in the data.

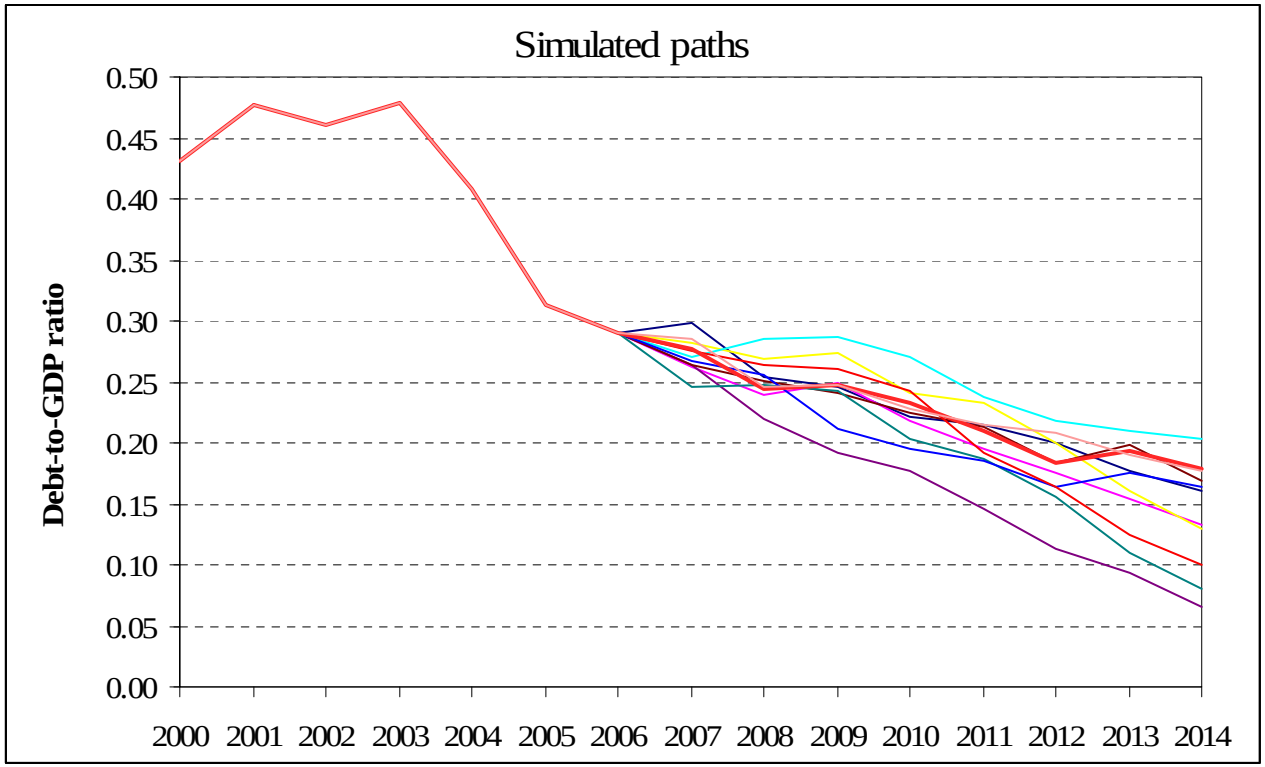
Fan Charts for DSA in a Nutshell

- Basic equation of motion for the debt as percentage of GDP:

$$d_t = \left[\frac{(1+r_t)}{(1+g_t)}\right] d_{t-1} - f_t \qquad \textbf{(A)}$$

- What you need is to generate multiple forecasts for the future trajectories of the debt ratios in order to build a “range” of outcomes.

Fan Charts for DSA in a Nutshell



Fan Charts for DSA in a Nutshell

- Basic equation of motion for the debt as percentage of GDP:

$$d_t = \left[\frac{(1+r_t)}{(1+g_t)}\right] d_{t-1} - f_t \qquad \textbf{(A)}$$

- What you need is to generate multiple forecasts for the future trajectories of the debt ratios in order to build a "range" of outcomes.
- One can then ask the following question: at the T-year horizon, what is the probability that the debt-to-GDP ratio be higher than "X" percent?
- The answer is simple: it is the ratio of the total number of paths that, at period T, end-up above the "X" percent threshold, over the total number of generated paths.

Fan Charts for DSA: generating the paths

- Option 1: RM Approach (Garcia and Rigobon, 2005 & Celasun et al. 2005)
  - The variables that enter (A) are stochastic and correlated.
  - Assume historical correlations –computed through suitable regression analysis—are likely to be relevant in the future.
  - Randomly generate a large number of shocks to all variables.
  - Associate with each shock a given debt path –following (A)—, much as in the IMF's DSA, except that each debt path now comes with a probability of occurrence.

Fan Charts for DSA: generating the paths

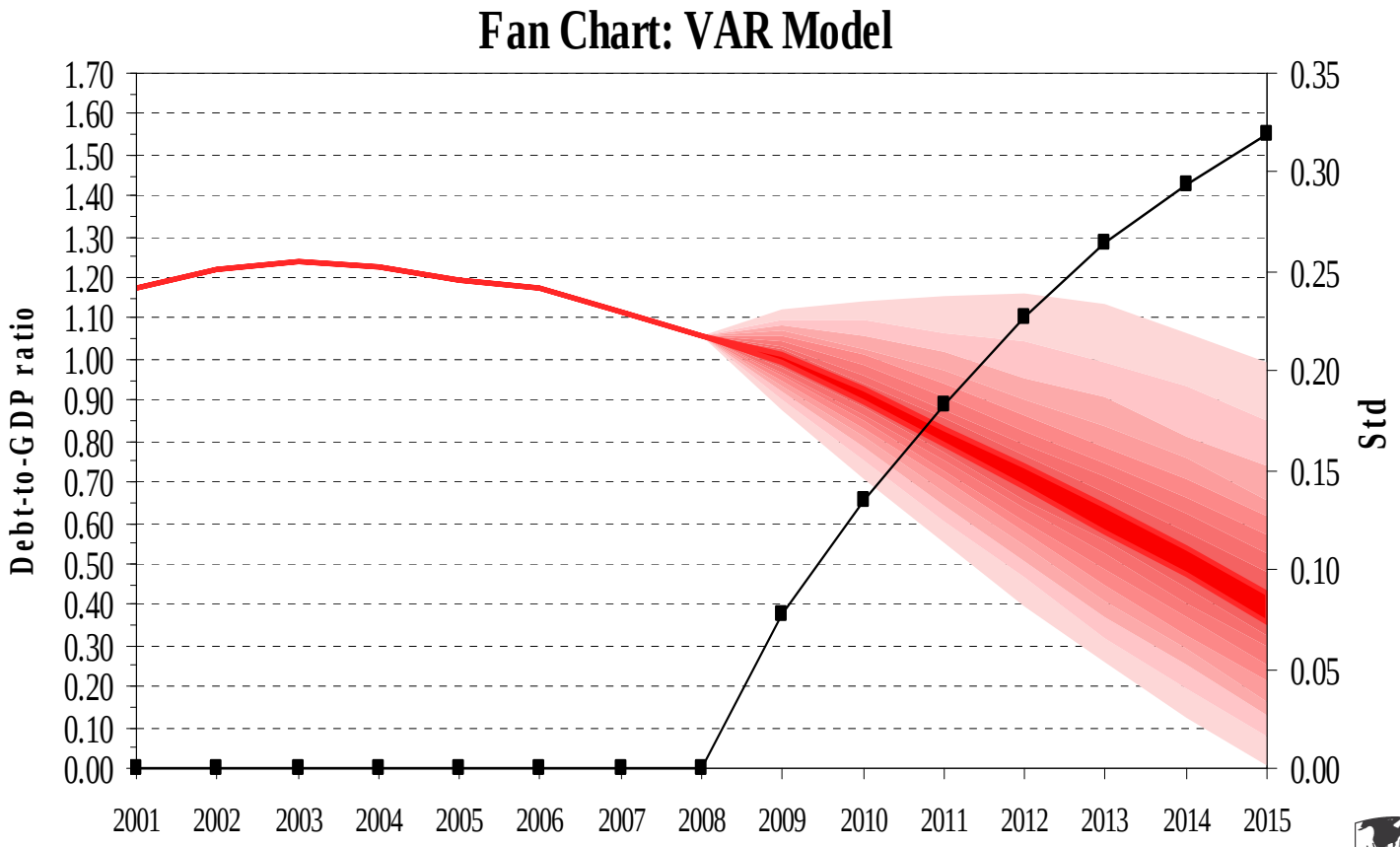
- Option 1: RM Approach
  - Vector Autoregressive Model (VAR):

$$Y_t = \mu_0 + \sum_{k=1}^{p} \mu_k Y_{t-k} + \xi_t \quad where\ Y_t = (r_t, g_t, f_t, \pi_t, e_t);\ \xi \sim N(0, \Omega)$$

  - μ_k is a vector of coefficients
  - Using the coefficients from the VAR we can compute the path of the variables in Y_t which can be used to estimate the "central" path of d_t.
  - Using the Choleski decomposition of the reduced form residuals matrix we generate multiple shocks to the variables of the debt equation and use them to obtain several paths for d_t.

Fan Chart for DSA: generating the paths

- Option 1: RM Approach



Fan Charts for DSA: generating the paths

- Option 2: External forecasts (Borensztein, Cavallo and Valenzuela 2007)
  - Like the IMF DSA, use forecasts for the variables that affect the evolution of the debt to trace a baseline of the resulting evolution of d_t –using (A)—.
  - Assume that historical variances of the variables that affect the evolution of the debt are likely to be relevant in the future.
  - Randomly generate a large number of shocks to all variables.
  - Associate with each shock a given debt path –following (A)—, much as in the IMF's DSA, except that each debt path now comes with a probability of occurrence.

Fan Charts for DSA

- Option 2: External Forecasts
  - The central projection is forward looking.

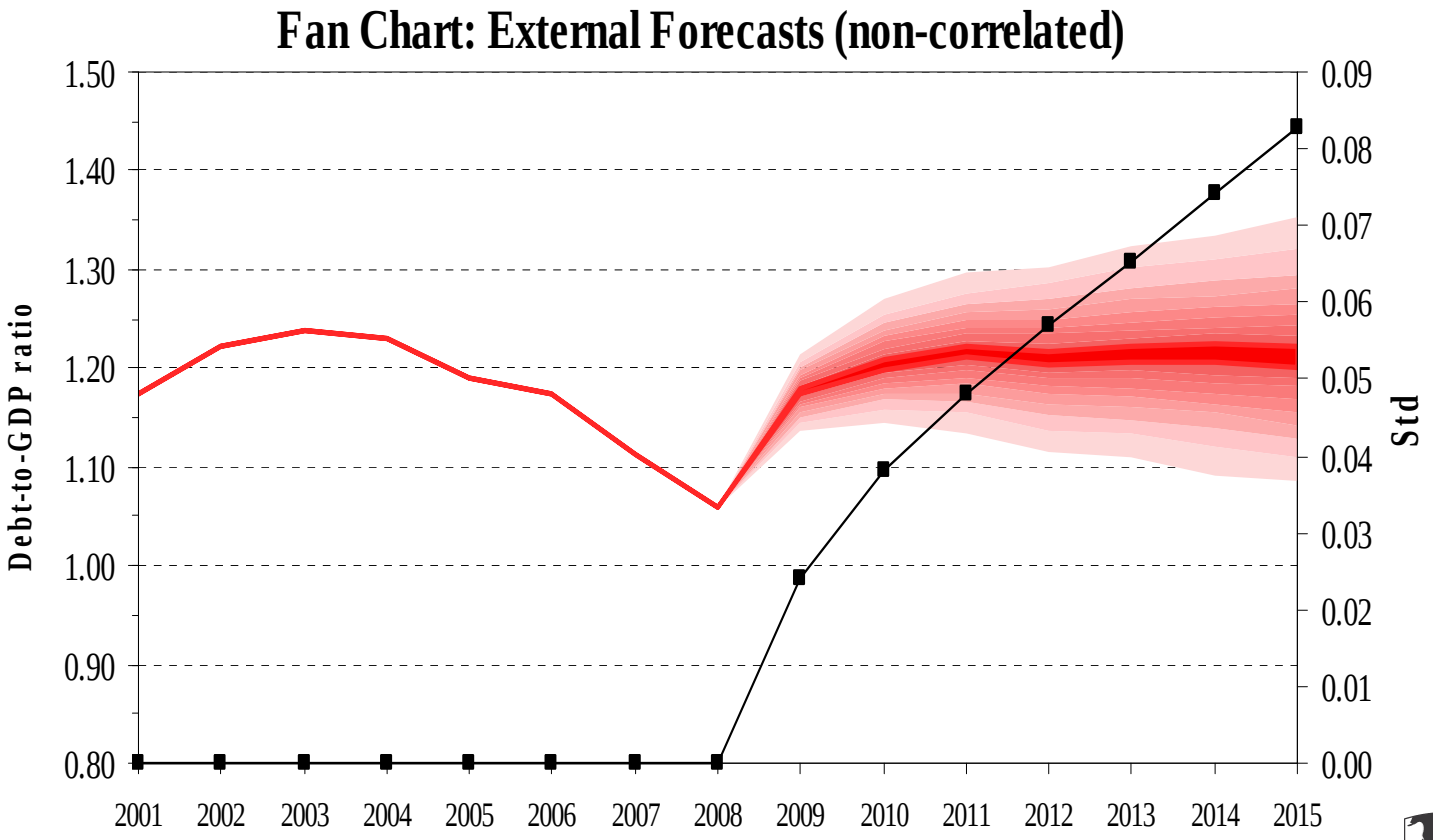
$$x_\tau^s = x_\tau^{Ext} + e_\tau \; ; \; for \; \tau \in [t+1, T]$$

$$where: \; e \sim N(0, \hat{\sigma}^2)$$

  - σ^2 is equal to the sample variance (historical)

Fan Charts for DSA: generating the paths

- Option 2: External Forecast



Fan Charts for DSA: generating the paths

- Option 3: External forecasts with correlated errors.
  - Like the IMF DSA, use forecasts for the variables that affect the evolution of the debt to trace a baseline of the resulting evolution of d_t –using (A)—.
  - Assume historical correlations –computed through suitable regression analysis—are likely to be relevant in the future.
  - Randomly generate a large number of shocks to all variables.
  - Associate with each shock a given debt path –following (A)—, much as in the IMF's DSA, except that each debt path now comes with a probability of occurrence.

Fan Charts for DSA: generating the paths

- Option 3: External forecasts with correlated errors.
  - The central projection is forward looking.
  - The correlation in the macro variables is accounted for

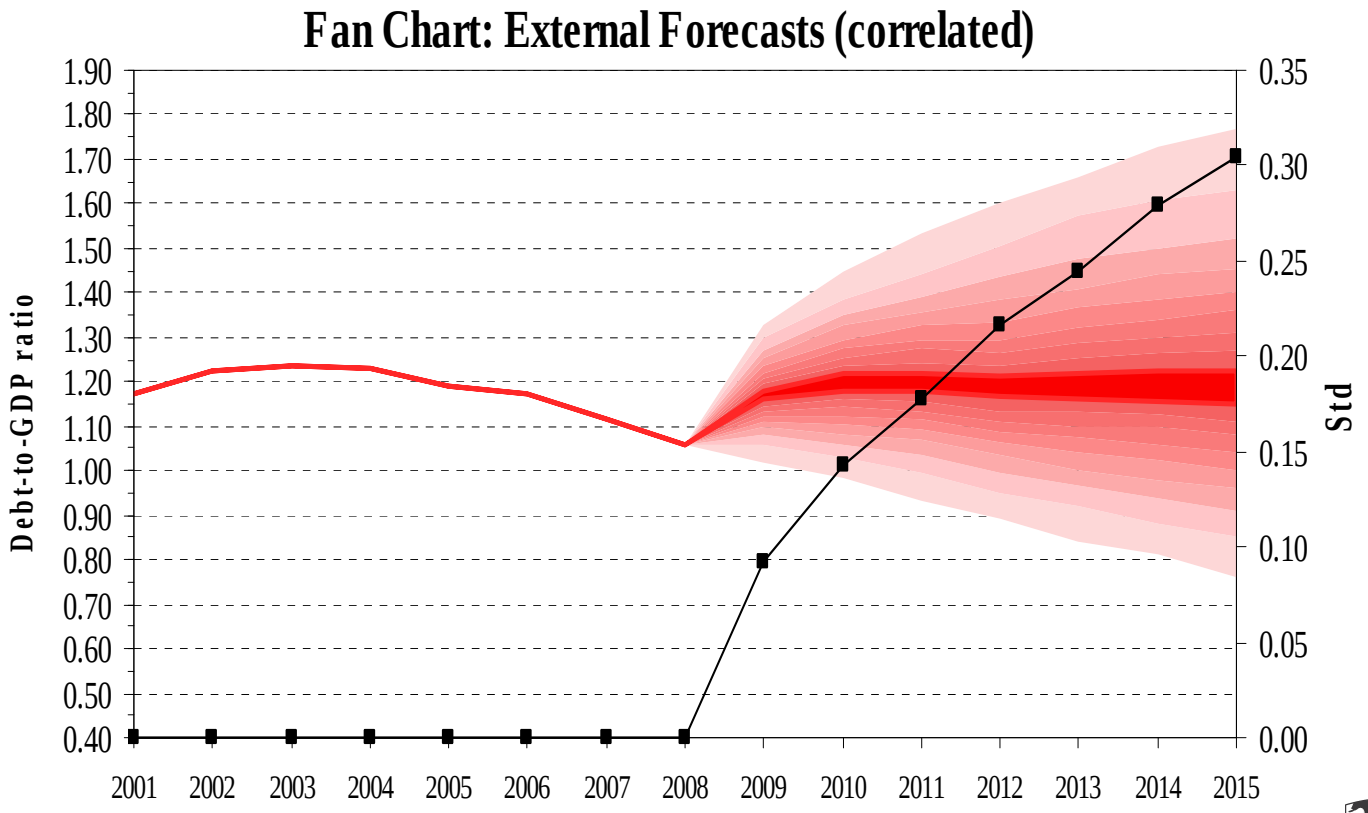
$$x_\tau^s = x_\tau^{Ext} + \eta_\tau \; ; \; for \; \tau \in [t+1, T]$$

$$where: \; \eta \sim N(0, \hat{\Omega})$$

  - Where $\hat{\Omega}$ is the var-cov matrix from a VAR.

Fan Charts for DSA: generating the paths

- Option 3: External forecast with correlated errors



Fan Charts for DSA: generating the paths

- Option 4: Weighted Forecast
  - A combination of options (1) and (3)

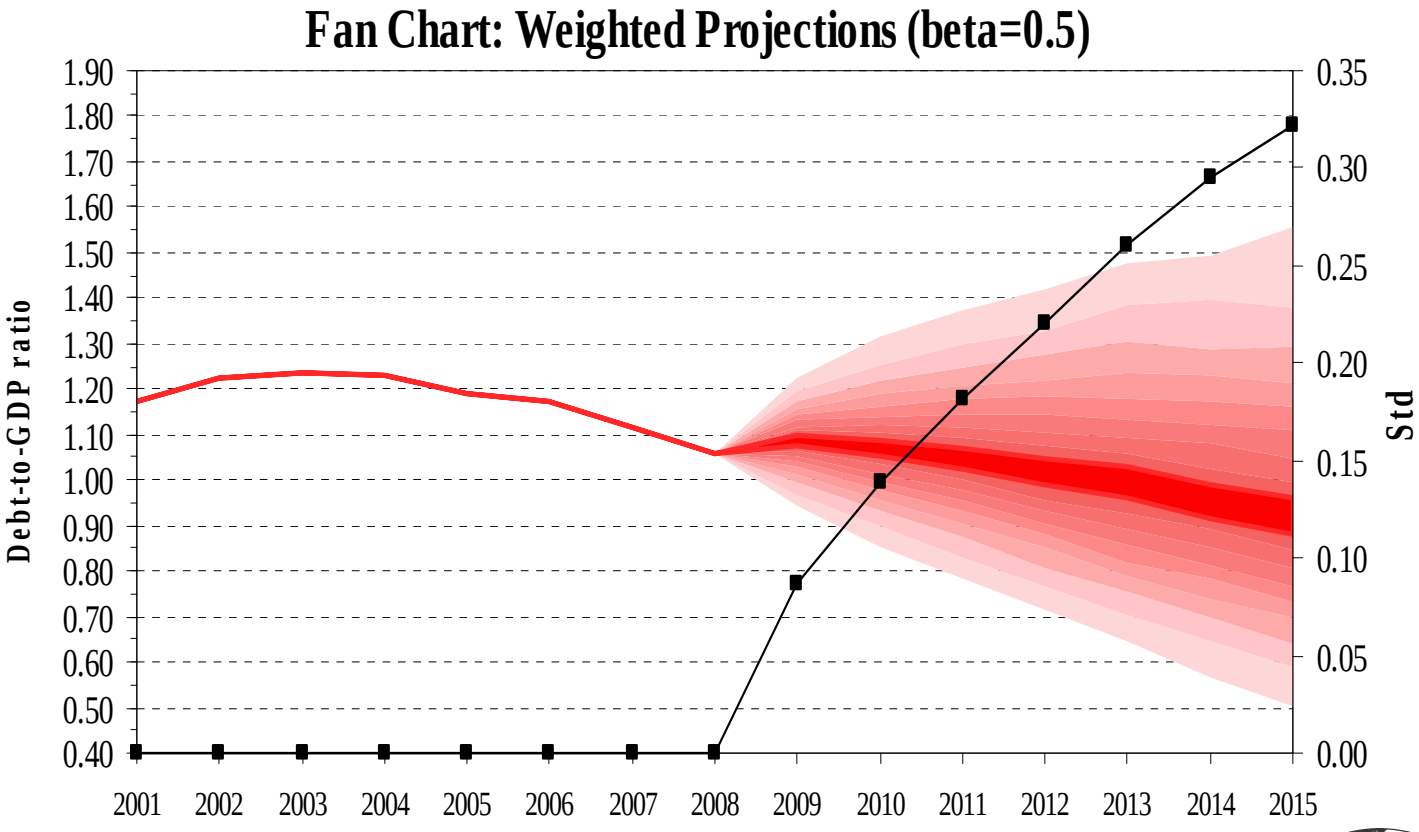
$$x_\tau^s = \beta * x_\tau^{Ext} + (1-\beta) * x_\tau^{VAR} + \eta_\tau \; ; \; for \; \tau \in [t+1, T]$$

$$where: \; \eta \sim N(0, \hat{\Omega})$$

  - Where $\hat{\Omega}$ is the var-cov matrix from a VAR.

Fan Charts for DSA: generating the paths

- Option 4: Weighted Forecast



Fan Charts for DSA: potential problems

- Caveats (Wyplosz, 2007):
  - All approaches to DSA have to rely on assumptions about the future evolution of budget balances, GDP, interest rates, etc. **The usefulness of the conclusions is directly related to the validity of these assumptions**, which by definition are neither safe nor testable.
  - Fan charts do provide some useful information, but the precision of these charts is unknown. **It depends on the quality of the data**, on the performance of the underlying econometric analysis and on the relevance of the history for the future.

Fan Charts for DSA: user's guide

- Step-by-step procedure:
  1. Define the debt accumulation equation to be used in the analysis

Fan Charts for DSA: user's guide

- Examples of debt accumulation equation:
  - Case 1: Fully dollarized economy (i.e., Ecuador)

$$d_t = \left(\frac{1+r^f}{1+g} \right) d_{t-1} - f_t$$

  - Case 2: Fully dollarized debt but the country keeps a domestic currency (i.e., Argentina pre-2002)

$$d_t = \left(\frac{(1+r^f)(1+\Delta e)}{1+g} \right) d_{t-1} - f_t$$

  - Case 3: Partially dollarized debt (i.e., Peru)

$$d_t = \left[\left(\frac{1+r^d}{1+g} \right) \alpha + \left(\frac{(1+r^f)(1+\Delta e)}{1+g} \right) (1-\alpha) \right] d_{t-1} - f_t$$

Fan Charts for DSA: user's guide

- Step-by-step procedure:
  1. Define the debt accumulation equation to be used in the analysis
  2. Select a time-horizon and a periodicity for the fan chart projections
  3. Collect historical series of data for the macro/financial variables included in the analysis
     - More variables that the ones that appearing in the debt equation can be included if the analyst consider that they can have non-negligible and relevant joint effects on the debt dynamics

Fan Charts for DSA: user's guide

- Step-by-step procedure:

4. Determine the methodology for the projection of the variables that establishes the debt dynamics. i.e: RMA, External Forecast, etc.

5. Simulation.

6. Evaluation of the methodology (see Arizala et al. for details).

Current work on DSA at the IDB

- Fan Charts are only one of a variety of DSA approaches that we use at the IDB.
- Other approaches used:
  - IMF DSA
  - The debt-stabilizing primary balance (Blanchard et. al. 1991; Buiter, 1985)
  - The Sudden Stop approach (Calvo, Izquierdo and Talvi 2005)
  - Natural debt-limit approach (Mendoza-Oviedo, 2004 & 2006)

Current work on DSA at the IDB

- We are currently in the process of testing a platform developed to standardize and run the different approaches using a single excel-template.
- Use a Excel Platform + Open Code in Visual Basic.

Data Input

	Nominal								GDP		
	Inflation	Foreign Inflation	Real Growth	Depreciation	i_ext	PS-to-GDP	i_dom	Debt-to-GDP	Current Accout	US$ curr	CCA/GDP
1981	5.20%	10.38%	4.4%	0.00%	17.33%			122.9%	-0.43	3.1	0.138792
1982	11.39%	6.16%	3.1%	0.00%	11.66%			128.0%	-0.54	3.6	0.150089
1983	18.42%	3.16%	4.2%	38.76%	6.50%	-4.98%	12.38%	194.7%	-0.30	3.2	0.093727
1984	28.87%	4.37%	0.9%	76.52%	14.76%	4.54%	13.29%	212.4%	-0.25	2.4	0.104171
1985	23.00%	3.53%	-0.9%	29.59%	11.80%	4.47%	19.03%	202.7%	-0.25	2.2	0.114746
1986	8.14%	1.94%	7.0%	-2.7%	9.9%	10.0%	20.9%	170.1%	-0.08	2.6	0.03
1987	4.72%	3.58%	7.7%	0.0%	9.8%	11.7%	18.2%	164.7%	-0.14	3.0	0.05
1988	24.14%	4.10%	-4.0%	0.0%	6.0%	10.4%	18.5%	146.8%	-0.06	3.5	0.02
1989	14.77%	4.79%	4.7%	3.6%	6.2%	17.8%	19.1%	137.3%	-0.32	4.1	0.08
1990	52.12%	5.42%	4.9%	26.3%	10.7%	17.4%	26.2%	128.7%	-0.57	5.2	0.11
1991	56.14%	4.22%	1.0%	68.1%	5.4%	15.5%	25.6%	178.1%	-0.20	4.9	0.04
1992	55.96%	3.04%	1.7%	90.1%	5.6%	13.5%	34.4%	115.6%	-0.13	4.1	0.03
1993	35.32%	2.97%	1.7%	0.0%	5.9%	9.8%	28.9%	119.6%	-0.06	5.6	0.01
1994	24.66%	2.60%	0.9%	3.5%	6.1%	11.3%	43.0%	101.2%	-0.07	6.8	0.01
1995	24.93%	2.81%	1.0%	66.5%	7.3%	10.4%	27.7%	96.9%	-0.25	5.1	0.05
1996	20.16%	2.94%	-1.1%	-12.0%	10.0%	6.1%	38.0%	79.6%	-0.25	6.9	0.04
1997	10.00%	2.34%	-1.0%	4.2%	6.3%	3.3%	21.1%	83.9%	-0.25	7.3	0.03
1998	8.59%	1.55%	-1.2%	2.0%	4.6%	8.8%	25.7%	85.1%	-0.17	7.6	0.02
1999	5.73%	2.19%	1.0%	11.4%	6.8%	12.9%	20.8%	87.3%	-0.30	7.3	0.04
2000	11.27%	3.37%	0.9%	4.1%	6.4%	14.7%	18.2%	88.7%	-0.40	7.9	0.05
2001	8.62%	2.82%	1.3%	7.0%	6.7%	9.5%	16.7%	117.4%	-0.67	8.1	0.08
2002	8.80%	1.60%	1.0%	5.3%	7.6%	7.2%	15.5%	122.2%	-0.95	8.5	0.11
2003	11.41%	2.30%	3.5%	19.3%	7.2%	11.0%	25.9%	123.8%	-0.62	8.2	0.08
2004	12.79%	2.67%	1.4%	6.0%	7.2%	11.9%	15.5%	122.9%	-0.56	8.8	0.06
2005	10.73%	3.38%	1.0%	1.8%	7.0%	11.5%	13.4%	119.1%	-0.92	9.7	0.09
2006	21.15%	3.22%	2.7%	5.6%	7.9%	7.1%	12.8%	117.5%	-1.17	11.5	0.10
2007	11.52%	2.86%	1.4%	4.9%	8.4%	7.1%	12.4%	111.3%	-1.84	12.4	0.15
2008	24.17%	3.80%	-1.2%	5.3%	8.1%	3.8%	15.0%	105.9%	-2.21	14.4	0.15
2009	11.17%	-0.94%	-2.6%	22.2%	7.8%	**6.0%**	20.7%	105.9%	-1.60	12.8	0.13
2010	9.10%	-0.08%	-0.3%	5.4%	7.6%	**6.0%**	21.0%	105.9%	-1.44	13.2	0.11
2011	8.29%	0.70%	1.5%	3.9%	7.4%	**5.9%**	21.0%	105.9%	-1.45	13.9	0.10
2012	7.17%	1.73%	2.7%	2.7%	7.2%	**5.8%**	20.0%	105.9%	-1.44	14.9	0.10
2013	6.24%	2.13%	2.2%	1.7%	7.2%	5.1%	18.0%	105.9%	-1.50	15.9	0.09
2014	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2015	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2016	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2017	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2018	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2019	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2020	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2021	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2022	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2023	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2024	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2025	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2026	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2027	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2028	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2029	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08
2030	6.05%	2.21%	2.1%	1.8%	7.2%	5.0%	16.0%	105.9%	-1.40	16.9	0.08248

Historical Data

Medium term-projections

Long-term projections

IDB

Data Output (i.e., Fan Charts)

Date	gdp	i_dom	rer	ps	Inf	i_ext	debt_gdp	Alpha
2001	0.0135	0.1671	0.0700	0.0946	0.0862	0.0666	1.1740	0
2002	0.0097	0.1554	0.0526	0.0720	0.0880	0.0762	1.2220	0
2003	0.0350	0.2594	0.1926	0.1099	0.1141	0.0719	1.2380	0
2004	0.0139	0.1547	0.0599	0.1186	0.1279	0.0720	1.2290	0
2005	0.0103	0.1339	0.0177	0.1147	0.1073	0.0696	1.1910	0
2006	0.0271	0.1279	0.0556	0.0710	0.2115	0.0789	1.1750	0
2007	0.0143	0.1237	0.0488	0.0709	0.1152	0.0842	1.1130	0
2008	-0.0118	0.1504	0.0526	0.0378	0.2417	0.0809	1.0593	0.451
2009	-0.0260	0.2071	0.2220	0.0602	0.1117	0.0777		0.4
2010	-0.0030	0.2100	0.0544	0.0604	0.0910	0.0758		0.3
2011	0.0152	0.2100	0.0393	0.0595	0.0829	0.0737		0.6
2012	0.0269	0.2000	0.0272	0.0579	0.0717	0.0717		0.4
2013	0.0224	0.1800	0.0175	0.0505	0.0624	0.0717		0.3
2014	0.0207	0.1600	0.0184	0.0498	0.0605	0.0717		0.5
2015	0.0207	0.1600	0.0184	0.0498	0.0605	0.0717		0.5

Probabilistic Assumptions: Standard Deviations		Betas
GDP growth	1	1
Interest Rate on Domestic Curr Debt	1	1
Exchange Rate	1	1
Primary Surplus	1	1
Inflation Rate	1	1
Interest Rate on Foreign Curr Debt	1	1
Initial Inputs of the Simulation		
Initial Level of Debt	1.0593	

GDP	-0.0118
Interest Rate on Domestic Curr Debt	0.1504
Exchange Rate	0.0526
Primary Surplus	0.0378
Inflation Rate	0.2417
Interest Rate on Foreign Curr Debt	0.0809

Length of the Simulations	
Number of Periods of Observations	8
Number of Periods of Simulations	7
Last observed year	2008
Alpha option	2
Alpha fixed	0.5
Initial Alpha	0.7
PS split	0.5

Assumptions Fan- Chart

Coefficients from VAR(1)

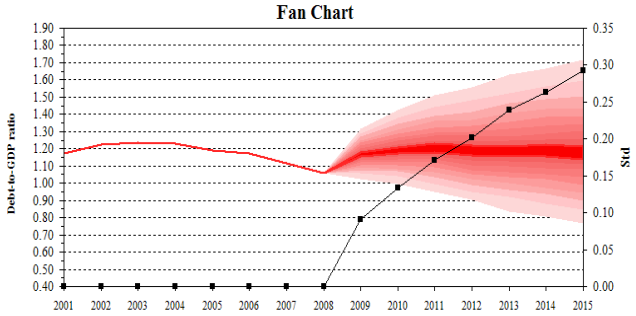
	r	e	g	f	inf	r_ext
r-1	0.381177	-0.222029	-0.075903	-0.078029	-0.362639	-0.043690
e-1	0.030412	-0.166870	-0.022860	-0.037750	-0.020116	0.035063
g-1	-0.005855	3.092071	-0.088928	-0.055835	2.005195	0.260979
f-1	0.070303	-1.494140	0.071355	0.479690	0.573265	-0.209077
inf-1	0.170649	1.314560	0.042331	0.061283	0.775004	-0.037679
r_ext_1	-0.514498	-2.156536	0.011254	-0.317684	-0.858691	0.058325
c	0.127694	0.239196	0.020299	0.090695	0.107584	0.099655

Var-Cov

	r	e	g	f	inf	r_ext
r	3.16E-03	-2.03E-03	-3.84E-05	2.12E-04	0.000695	0.0000101
e	-2.03E-03	0.0475	-0.000104	0.001138	0.01069	0.000943
g	-3.84E-05	-0.0001	0.000597	3.33E-04	-0.00043	0.000187
f	2.12E-04	0.001138	3.33E-04	7.93E-04	1.45E-04	0.0000382
inf	0.000695	0.01069	-0.00043	1.45E-04	0.006739	0.000432
r_ext	0.0000101	0.000943	0.000187	3.82E-05	0.000432	0.000241

Run

Proyection year	Shock to Debt
2001	
2002	
2003	
2004	
2005	
2006	
2007	
2008	
2009	
2010	
2011	
2012	
2013	
2014	
2015	



Value at Risk of the Debt / GDP	
Threshold of the debt limit (X)	1.11
Prob(Debt/ GDP) > X	0.67

Sensitivity Analysis

X	Prob(Debt to GDP)>X
>1.11	0.67
>1.16	0.56
>1.21	0.44
>1.26	0.34

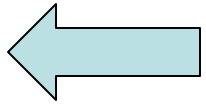
If alpha option =1

Min alpha (last simulation year)	0.45
Max alpha (last simulation year)	1.01
Mean alpha (last simulation year)	0.79

IDB

Inter-American Development Bank / www.iadb.org

- Risk Profile Developed Country:
  - Recessions are usually accompanied by a decrease in the interest rate (i.e., expansionary monetary policies). There is an automatic stabilizer.
- Risk Profile for Developing Country:
  - Recessions deteriorate fiscal accounts (even without discretionary expansionary fiscal policies), increases the real interest rate, induces inflation and depreciates the exchange rate. There is an automatic de-stabilizer.



IDB

Additional Background

- The fan charts methodology is part of what is known as *probabilistic approaches* to uncertainty analysis.
- Uncertainties are characterized by the probabilities associated with events or outcomes of a set of variables.
- The probability of an outcome can be interpreted in terms of the frequency of occurrence of any of those possible results.
- The forecaster should assign a given probability to each of the outcomes for a given forecasting horizon. Options:
  - Ex-ante probability distribution.
  - A given theoretical model and posterior simulation.
  - Statistical methods followed by simulations.
  - Subjective probability judgments.
- Likewise, these options can be combined.

Additional Background