

Long-duration Bonds and Sovereign Defaults*

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Abstract

This paper extends the baseline framework used in recent quantitative studies of sovereign default by assuming that the government can borrow using long-duration bonds. This contrasts with previous studies, which assume that the government can borrow using bonds that mature after one quarter. We solve the model assuming that the government issues bonds with a duration similar to the average duration of sovereign bonds in emerging economies and show that the model is able to generate a substantially higher and more volatile interest rate. This narrows the gap between the predictions of the model and the data, which indicates that the introduction of long-duration bonds may be a useful tool for future research about emerging economies. Our analysis is also relevant for the study of other credit markets.

JEL classification: F34, F41.

Keywords: Sovereign Default, Endogenous Borrowing Constraints, Bond Duration, Debt Dilution, Markov Perfect Equilibrium.

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1 Introduction

Business cycles in small emerging economies differ from those in developed economies. Emerging economies feature interest rates that are higher, more volatile and countercyclical (interest rates are usually acyclical in developed economies), and have higher output volatility, higher volatility of consumption relative to income, and more countercyclical net exports (see Aguiar and Gopinath (2007), Neumeyer and Perri (2005), and Uribe and Yue (2006)). Due to the high volatility and countercyclicality of the interest rate in emerging economies, a state-dependent interest rate schedule is a key ingredient in any model designed to explain the cyclical behavior of aggregate quantities and prices in these economies. In this respect, some studies assume an exogenous interest rate schedule.¹ Others present models with microfoundations for the interest rate schedule based on the risk of default. This is the approach taken by recent quantitative studies of sovereign default, which extend the framework proposed by Eaton and Gersovitz (1981).² The setup studied in our paper belongs to the second class of models.

As in previous studies of sovereign default, we analyze a small open economy that receives a stochastic endowment stream of a single tradable good. The objective of the government is to maximize the expected utility of private agents. The government makes two decisions in every period. First, it decides whether to default on previously issued debt. The cost of defaulting is represented by an endowment loss that is incurred in the default period. Second, the government decides how much to borrow or save. The government can borrow (save) by issuing (buying) non-contingent bonds that are priced in a competitive market inhabited by a large number of identical risk-neutral lenders. Lenders have perfect information regarding the economy's endowment.

The main difference between our paper and previous studies is that while previous quantitative studies assume that sovereign bonds mature after one quarter, we assume that the govern-

¹See, for instance, Aguiar and Gopinath (2007), Neumeyer and Perri (2005), Schmitt-Grohé and Uribe (2003), and Uribe and Yue (2006).

²See Aguiar and Gopinath (2006), Arellano (2008), Arellano and Ramanarayanan (2006), Bai and Zhang (2006), Benjamin and Wright (2008), Cuadra and Saprizza (2006, 2008), D'Erasmus (2008), Eyigungor (2006), Hatchondo et al. (2006, 2007, 2008), Lizarazo (2005, 2006), Mendoza and Yue (2007), and Yue (2005). These models share blueprints with the models used in quantitative studies of household bankruptcy—see, for example, Athreya (2002), Athreya et al. (2007a,b), Chatterjee et al. (2007a), Chatterjee et al. (2007b), Li and Sarte (2006), Livshits et al. (2008), and Sánchez (2008).

ment issues bonds that pay an infinite stream of coupons until a default is declared. The coupon payments promised decrease at a constant rate. This assumption allows us to introduce debt instruments with long duration in a simple and tractable way: the number of state variables is independent of the duration of debt.

We solve the model assuming that the government issues bonds that have a duration of four years—similar to the average duration in emerging economies, as documented by Cruces et al. (2002) and Cunningham et al. (2001)—and show that the predictions of the model change significantly when one-quarter bonds are replaced by four-year bonds. The mean annual spread—extra yield over the risk-free rate—increases from 0.1% in the model with one-quarter bonds to 2.5% in the model with four-year bonds. The spread volatility is also significantly higher in the model with four-year bonds: it increases from 0.03% to 0.23%. The model with long-duration bonds is also able to replicate other salient features of emerging economies, like the excessive volatility of consumption compared to GDP and the countercyclicality of the trade balance. Thus, our results narrow the gap between the predictions of the baseline model of sovereign default and the data, which indicates that introducing long-duration bonds may be a useful tool for future research about emerging economies. For instance, the modeling strategy undertaken in this paper could simplify the study of the optimal maturity structure of sovereign debt (see, for example, Alfaro and Kanczuk (2007), Arellano and Ramanarayanan (2006), Bi (2006), and Broner et al. (2007)), and the study of the costs of debt dilution (see, for instance, Bi (2006), Bizer and DeMarzo (1992), Detragiache (1994), Hatchondo and Martinez (2007), Kletzer (1984), Sachs and Cohen (1982), and Tirole (2002)).

The rest of the article proceeds as follows. Section 2 introduces the model. Section 3 presents a recursive formulation. Section 4 discusses the parameter values used to solve the model. Section 5 presents the results. Section 6 concludes.

2 The model

The basic framework studied in this paper follows previous work that extends the model of sovereign default presented by Eaton and Gersovitz (1981) to study its quantitative performance.

The main difference between the model we present and the baseline framework is that our model allows us to study long-duration bonds.

2.1 The environment

There is a single tradable good. The economy receives a stochastic endowment stream of this good, where

$$\log(y_t) = (1 - \rho) \mu + \rho \log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$, and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

The objective of the government is to maximize the present expected discounted value of future utility flows of the representative agent in the economy, namely

$$E \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right],$$

where β denotes the subjective discount factor and the utility function is assumed to display a constant coefficient of relative risk aversion, denoted by σ . That is,

$$u(c) = \frac{c^{(1-\sigma)} - 1}{1 - \sigma}.$$

The government makes two decisions in each period. First, it decides whether to default. A default implies that the government repudiates all current and future debt obligations contracted in the past. The cost of a default is represented by the loss of an amount $\phi(y)$ of the endowment level realized in the default period. Second, the government decides the amount of bonds that are going to be purchased or issued in the current period.

We assume that a bond issued in period t promises an infinite stream of coupons, which decrease at a constant rate δ . In particular, a bond issued in period t promises to pay one unit of the good in period $t + 1$ and $(1 - \delta)^{s-1}$ units in period $t + s$ with $s \geq 2$.

As in previous studies of sovereign default, we assume that each period the government can choose any issuance amount for that period, anticipating that the issuance price is such that

lenders buying bonds make zero profits in expectation.³ Lenders can borrow or lend at the risk-free rate r , and have perfect information regarding the economy's endowment.

Following recent quantitative studies of default risk, we assume that the government cannot commit to future default and borrowing decisions. Thus, one may think about our environment as a game in which the government making the default and borrowing decisions in period t is a player who takes as given the default and borrowing strategies of other players (governments) who will decide after t . In contrast, if the government could commit to future default and borrowing decisions, it would internalize the effect of period- t decisions in every period before t . We use the Markov Perfect Equilibrium concept. That is, we assume that in each period, the government's equilibrium default and borrowing decisions depend only on payoff relevant state variables.

2.2 Discussion of assumptions

First, note that the coupon structure assumed in the paper allows us to consider debt with duration longer than one period without increasing the dimensionality of the state space. The government default and issuance decisions in period t are influenced by its debt obligations in the current period and in every future period. Our assumptions imply that the amount of coupon obligations that are due today is a unique state variable that allows us to keep track of all government's future payment obligations. Since all bonds promise a sequence of coupons that decrease at the same constant rate, current coupon obligations are a sufficient statistic to predict the stream of future debt obligations derived from issuances in all previous periods.

In contrast, alternative strategies to model long-duration bonds could imply an increase in the dimensionality of the state space. For instance, suppose that the government issues zero coupon bonds that mature n periods ahead. In period t , the default and issuance decisions would depend on the bonds issued between period $t - n$ and period $t - 1$, which determine the

³There are several borrowing games that would lead to government's borrowing opportunities as the ones described above. For instance, it could be assumed that each period, the government conducts the following auction: First, the government announces the size of the current bond issuance. Then, each lender offers the government a price at which he is willing to buy the bonds the government is issuing or to sell the bonds the government wants to buy. The government then chooses the lenders with whom it will perform the transaction. Finally, the transaction is performed and the current-period borrowing game ends.

government's debt obligations from period t to period $t + n - 1$. Thus, in general, in order to solve the model, one would have to keep track of n state variables. Similarly, if one assumes that bonds promise a infinite sequence of coupons that decrease at a constant rate, but allows this rate to be different for bonds issued in different periods, one would have to keep track of bond issuances in all previous periods, and it would not be possible to summarize all future payment obligations with unique state variable.

It should also be mentioned that in our framework the stock of next-period coupon obligations can easily be determined using the stock of current-period obligations, the current-period default decision, and current-period issuances. Let b (b') denote the amount of outstanding coupon claims at the beginning of the current (next) period. A negative value of b implies that the government was a net issuer of bonds in the past. Let d denote the current-period default decision. We assume that d is equal to 1 if the government has defaulted in the current period and is equal to 0 if it has not. Let ι denote the current-period issuance level. Then,

$$b' = b(1 - \delta)(1 - d) - \iota$$

presents the law of motion for coupon payment obligations. Note that this law of motion resembles the law of motion for capital in the neoclassical growth model. If, for example, the government chooses $b' = (1 - d)(1 - \delta)b$, it is neither issuing nor buying bonds. That is, the amount of coupons that will mature next period are solely determined by past debt issuances and the current default decision. If the government chooses $b' < (1 - d)(1 - \delta)b$, it is issuing new bonds, and if $b' > (1 - d)(1 - \delta)b$, the government is purchasing bonds.

The assumed coupon structure also allows us to calibrate the bond duration—how long it takes for the price of a bond to be repaid by its internal cash flows—which in our framework depends on δ . In this paper, we shall use the Macaulay definition of duration.⁴ According to this definition, the duration of a bond is equal to the weighted sum of future payment dates, where

⁴Empirical studies and credit rating agencies use measures of duration of sovereign bonds based on this definition. See, for example, Cruces et al. (2002) and Cunningham et al. (2001). Copeland and Weston (1992) present a thorough discussion of the concept of bond duration.

each date is weighted by the relative importance of the payments due in that date:

$$D \equiv \sum_{s=1}^{\infty} s \frac{C_s (1 + r^*)^{-s}}{q},$$

where D denotes the duration of a bond that promises a coupon C_s at date s and has a price q , and r^* denotes the constant per-period yield delivered by the bond. According to this definition, the coupon structure assumed in this paper implies that the duration of a bond is given by

$$D = \frac{1 + r^*}{\delta + r^*}. \quad (1)$$

Equation (1) shows that one can easily calibrate δ to obtain a desired bond duration. Note also that the one-period-bond model is a particular case of our framework: It corresponds to $\delta = 1$, which implies a duration of one period according to equation (1).

The coupon structure assumed in the paper can be interpreted as if the debt issued by the government consisted of a portfolio of zero-coupon bonds of different maturity, where the portfolio weights decline geometrically with maturity. It should be emphasized that in our model the maturity structure of sovereign debt is fixed and cannot be changed by the government. Our goal is to show that abandoning the one-period-bond assumption can enhance the ability of the model to explain the behavior of interest rates in emerging economies. Analyzing how the government chooses the optimal maturity structure of sovereign debt is beyond the scope of this paper.

Although we assume perpetual bonds because this allows us to have a tractable framework, it should be mentioned that perpetual bonds are commonly used mainly by banks but also by some governments. The best known examples of perpetual government bonds are the consols issued by the British government in the Eighteenth Century. Other examples of perpetual sovereign bonds exist but in general, issuances of perpetual government bonds are rare.

The assumption that defaults trigger an output loss is standard and intends to capture the disruptions in economic activity caused by a default decision. It has been argued that a sovereign default increases the borrowing cost of domestic firms and, thus, it reduces output. Using micro-level data, Arteta and Hale (2006) find that sovereign debt crises are systematically accompanied

by a large decline in foreign credit to domestic private firms. This could be the case because a sovereign default may induce investors to believe that there is a higher risk of expropriation or bad economic conditions, and therefore, it may reduce firms' net worth and their ability to borrow (see Sandleris (2006) and the references therein). IMF (2002), Kumhof (2004), and Kumhof and Tanner (2005) discuss how financial crises that lead to severe recessions follow a sovereign default. Similarly, Kaminsky and Reinhart (1999) show that currency devaluations in developing countries tend to cause banking problems. Kobayashi (2006) presents a model in which a shock that disturbs the payments system causes a decrease in aggregate productivity. Mendoza and Yue (2007) study the link between sovereign-default risk and output.

We do not assume that countries can be excluded from capital markets after a default episode. Wright (2005) discusses how in the past three decades the sovereign debt market has become more competitive and explains how an increase in competition (number of creditors) may diminish the creditors' ability to coordinate and punish defaulting countries by excluding them from capital markets (see also Athreya and Janicki (2006), Cole et al. (1995), Hatchondo et al. (2007) and Wright (2002)). Hatchondo et al. (2008) discuss how defaults and difficulties in market access that follow a default can be jointly explained by the presence of political turnover. Empirical studies suggest that once variables such as the quality of policies and institutions are used as controls, market access is not significantly influenced by past default decisions (see, for example, Eichengreen and Portes (2000), Gelos et al. (2004), and Meyersson (2006)). Hatchondo et al. (2007) show that eliminating the exclusion punishment from a baseline model of sovereign default does not affect significantly the mean spread and the spread volatility generated by the model.

3 Recursive formulation

Let $V(b, y)$ denote the government's value function at the beginning of a period, that is, before the default decision is made. Let $\tilde{V}(d, b, y)$ denote its value function after the default decision has been made. We use x' to denote the next-period value of a variable x . Let $F(y' | y)$ denote the conditional cumulative distribution function of the endowment in the next period. For any bond price function $q(b', y)$, the function $V(b, y)$ satisfies the following functional equation:

$$V(b, y) = \max_{d \in \{0,1\}} \{d\tilde{V}(1, b, y) + (1-d)\tilde{V}(0, b, y)\}, \quad (2)$$

where

$$\tilde{V}(d, b, y) = \max_{b' \in \mathbb{R}} \left\{ u(c) + \beta \int V(b', y') F(dy' | y) \right\}, \quad (3)$$

and

$$c = y - d\phi(y) + (1-d)b - q(b', y) [b' - (1-d)(1-\delta)b].$$

The bond price that satisfies the lenders' zero-profit condition is given by the following functional equation:

$$q^{ZP}(b', y) = \frac{1}{1+r} \int [1 - h(b', y')] F(dy' | y) + \frac{1-\delta}{1+r} \int [1 - h(b', y')] q^{ZP}(g(h(b', y'), b', y'), y') F(dy' | y), \quad (4)$$

where $h(b, y)$ and $g(d, b, y)$ denote the default and borrowing rules that lenders expect the government to follow in the future. The default rule $h(b, y)$ is equal to one if the government defaults, and is equal to zero otherwise. The function $g(d, b, y)$ determines the value of coupons that will mature next period. Note that the government computes its current default and borrowing decisions taking as given its future default and borrowing decisions, that is, $h(b, y)$ and $g(d, b, y)$. In equilibrium, the optimal default and borrowing rules that solve problems (2) and (3) must be equal to $h(b, y)$ and $g(d, b, y)$ for all possible values of the state variables.

The first term in the right-hand side of equation (4) equals the expected value of the next-period coupon payment promised in a bond. The second term in the right-hand side of equation (4) equals the expected value of all other future coupon payments, which is summarized by the expected price at which the bond could be sold next period. Note that the second term is equal to zero with one-period bonds ($\delta = 1$).

Definition 1 *A Markov Perfect Equilibrium is characterized by*

1. a set of value functions $\tilde{V}(d, b, y)$ and $V(b, y)$,
2. a default decision rule $h(b, y)$ and a borrowing rule $g(d, b, y)$,
3. and a bond price function $q^{ZP}(b', y)$,

such that:

(a) given $h(b, y)$ and $g(d, b, y)$, $V(b, y)$ and $\tilde{V}(d, b, y)$ satisfy functional equations (2) and (3) when the government can trade bonds at $q^{ZP}(b', y)$;

(b) given $h(b, y)$ and $g(d, b, y)$, the bond price function $q^{ZP}(b', y)$ offered to the government satisfies the lenders' zero-profit condition implicit in equation (4);

(c) the default rule $h(b, y)$ and borrowing rule $g(d, b, y)$ solve the dynamic programming problem defined by equations (2) and (3), when the government can trade bonds at $q^{ZP}(b', y)$.

As discussed by Krusell and Smith (2003), typically, there is a problem of multiplicity of Markov perfect equilibria in infinite-horizon economies. In order to avoid this problem, we analyze the equilibrium that arises as the limit of the finite-horizon-economy equilibrium.

4 Parameterization

The model is solved numerically using value function iteration and spline interpolation.⁵ Table 1 presents the parameterization used in this paper.

We assume a coefficient of relative risk aversion of 2, which is within the region of accepted values in studies of real business cycles. A period in the model refers to a quarter. The risk-free interest rate is set equal to 1%. As in Hatchondo et al. (2008), the parameter values that govern the endowment process are chosen so as to mimic the behavior of GDP in Argentina from the fourth quarter of 1993 to the third quarter of 2001. The parameterization of the output process is similar to the parameterization used in other studies that consider a longer sample period (see Aguiar and Gopinath (2006)).

As in previous studies (see, for example, Aguiar and Gopinath (2006)), we assume that the

⁵The algorithm finds two value functions, $\tilde{V}(1, b, y)$ and $\tilde{V}(0, b, y)$. Convergence in the equilibrium price function $q(b', y)$ in equation (4) is also assured.

Risk aversion	σ	2
Interest rate	r	1%
Output autocorrelation coefficient	ρ	0.9
Standard deviation of innovations	σ_ϵ	2.7%
Mean log output	μ	$(-1/2)\sigma_\epsilon^2$
Output loss	λ	8.3%
Discount factor	β	0.95

Table 1: Parameter values.

loss in output triggered by a default is represented by a function $\phi(y) = \lambda y$. The value of λ is taken from Hatchondo et al. (2007).⁶ We have solved the model for different values of λ and found that the value of λ does not affect the main conclusion of the paper: The spread is higher and more volatile when one assumes a duration of sovereign bonds similar to the one observed in emerging economies instead of one-quarter bonds.

The discount factor is relatively low but higher than what is assumed in previous studies with a cost of defaulting similar to the one assumed in this paper (for instance, Aguiar and Gopinath (2006) assume $\beta = 0.8$). Low discount factors may be a result of political polarization in emerging economies (see Amador (2003) and Cuadra and Saprizza (2008)).

We present results for two values of δ . When $\delta = 1$, bonds have a duration of one quarter, which is the case analyzed in previous studies. When $\delta = 0.053125$, bonds have a duration of four years if future coupon payments are discounted at the risk-free rate r (the duration would be 3.66 years if future coupon payments were discounted using the average yield implicit in bond prices observed in equilibrium). Cruces et al. (2002) report that the average duration of Argentinean bonds included in the EMBI index was 4.13 years in 2000. This number is not significantly

⁶They find that in a baseline model of sovereign default, a value of λ equal to 8.3% delivers a default cost that is equivalent to the cost implied by the process of output loss assumed in Aguiar and Gopinath (2006) (a loss of 2% per period for an average of 10 periods). It should be mentioned that the calculation in Hatchondo et al. (2007) is made with a discount factor lower than the one we assume here.

different from what is observed in other emerging economies. Using a sample of 27 emerging economies, they find an average duration of 4.77 years with a standard deviation of 1.52.

5 Results

This section focuses on the ability of the model to replicate some stylized facts about the macroeconomic behavior of emerging economies. We simulate the model for a number of periods that allows us to extract 500 samples of 32 consecutive periods before a default. We focus on samples of 32 periods because we want to compare the artificial data generated by the model with Argentine data from the fourth quarter of 1993 to the third quarter of 2001. This is the same period that is considered in Hatchondo et al. (2008). The macroeconomic behavior observed over that period displays the same qualitative features that can be observed using a longer sample period or using data from other emerging markets (see, for example, Aguiar and Gopinath (2007), Neumeyer and Perri (2005), and Uribe and Yue (2006)). The only exception is that in the period we consider, the volatility of consumption is slightly lower than the volatility of income, while emerging market economies tend to display a higher volatility of consumption relative to income. Finally, in order to facilitate the comparison with the data, we only consider sample paths where the last default was declared at least two periods before the beginning of each sample.

Table 2 reports moments in the data and in our simulations.⁷ The moments reported in the table are chosen so as to evaluate the ability of the model to replicate distinctive business cycle properties of emerging economies. Relative to developed economies, emerging economies feature higher, more volatile and countercyclical interest rates; higher volatility of consumption relative to income; and more countercyclical net exports. The trade balance (TB) is expressed as a fraction of output (Y). The interest rate spread (R_s) represents the margin of extra yield over the risk-free rate and is expressed in annual terms.⁸ The logarithm of income and consumption

⁷The data for output, consumption, and trade balance was obtained from the Argentinean Finance Ministry. The spread before the first quarter of 1998 is taken from Neumeyer and Perri (2005), and from the EMBI Global after that.

⁸Let r^* denote the per-period constant yield implied by a bond price $q^{ZP}(b', y)$, namely

$$r^* = \frac{1}{q^{ZP}(b', y)} - \delta.$$

are denoted by y and c , respectively. The standard deviation of x is denoted by $\sigma(x)$ and is reported in percentage terms. The correlation between x and z is denoted by $\rho(x, z)$. Moments are computed using detrended series. Trends are found using the Hodrick-Prescott filter with a smoothing parameter of 1,600.

	Data	One quarter	Four years
$\sigma(y)$	3.17	3.15	3.07
$\sigma(c)$	2.98	3.24	3.14
$\sigma(TB/Y)$	1.35	0.17	0.12
$\sigma(R_s)$	2.51	0.03	0.23
$\rho(c, y)$	0.97	1.00	1.00
$\rho(TB/Y, y)$	-0.69	-0.46	-0.56
$\rho(R_s, y)$	-0.65	-0.94	-0.86
$\rho(R_s, TB/Y)$	0.56	0.73	0.81
$E(R_s)$	7.44	0.11	2.54

Table 2: Business cycle statistics. The second column is computed using data from Argentina from 1993 to 2001. The third (fourth) column reports the mean value of each moment over the first 500 samples of 32 periods before a default in an economy where the government issues bonds with a duration of one quarter (four years).

Table 2 shows that the mean annual spread increases from 0.1% with one-quarter bonds to 2.5% with four-year bonds. The spread volatility is also significantly higher in the model with four-year bonds; it increases from 0.03% to 0.23%. The table also shows that introducing four-year bonds does not damage the ability of the model to replicate other features of the data: Other statistics reported in the table are similar when computed with one-period bonds and four-year bonds. Differences in the behavior of the spread are also apparent in Figure 1, which shows the combinations of income realizations and spread levels chosen in equilibrium in the simulations of the model with one-quarter and four-year bonds.

Figure 2 illustrates how the model generates higher and more volatile spreads with four year

The annualized spread is given by $R_s = \left(\frac{1+r^*}{1+r}\right)^4 - 1$.

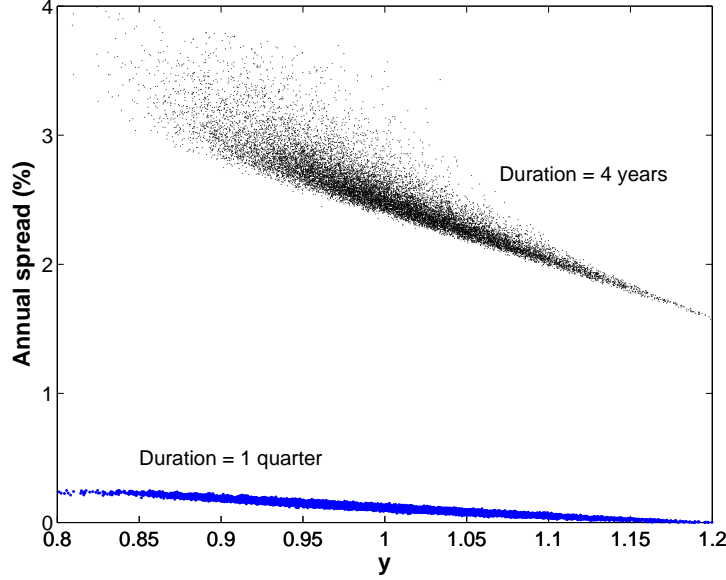


Figure 1: Endowment realizations and equilibrium spread levels in the economies with one-quarter and four-year bonds. The plot shows 500 samples of 32 periods before a default episode.

bonds. It presents spread yields that render lenders zero profits in expectation as a function of the next-period debt level. The spread functions in Figure 2 represent the combinations of next-period debt levels and annual spread yields that the government could choose. The figure also presents the government's optimal choices. In order to facilitate the comparison across different bond durations, we plot the spread instead of the bond price, and the face value—defined as the present value of future payment obligations discounted at the risk-free rate—of next-period debt $\frac{b'}{\delta+r}$ instead of b' .

In order to shed light on the government's optimal choices illustrated by the solid dots in Figure 2, we discuss next the Euler equation that determines these choices. In order to simplify the notation, we do not write consumption, default, and future borrowing as functions of the state variables. Let $\tilde{b} \equiv (1-d)(1-\delta)b$ denote the interim value of next-period coupon obligations. We use $f_i(x_1, \dots, x_n)$ to denote the first-order derivative of the function f with respect to the argument x_i . The Euler equation is given by

$$u_1(c)q^{ZP}(b', y) = \beta \int u_1(c') (1-d') [1 + q^{ZP}(b'', y')(1-\delta)] F(dy' | y) - u_1(c)q_1^{ZP}(b', y)(b' - \tilde{b}). \quad (5)$$

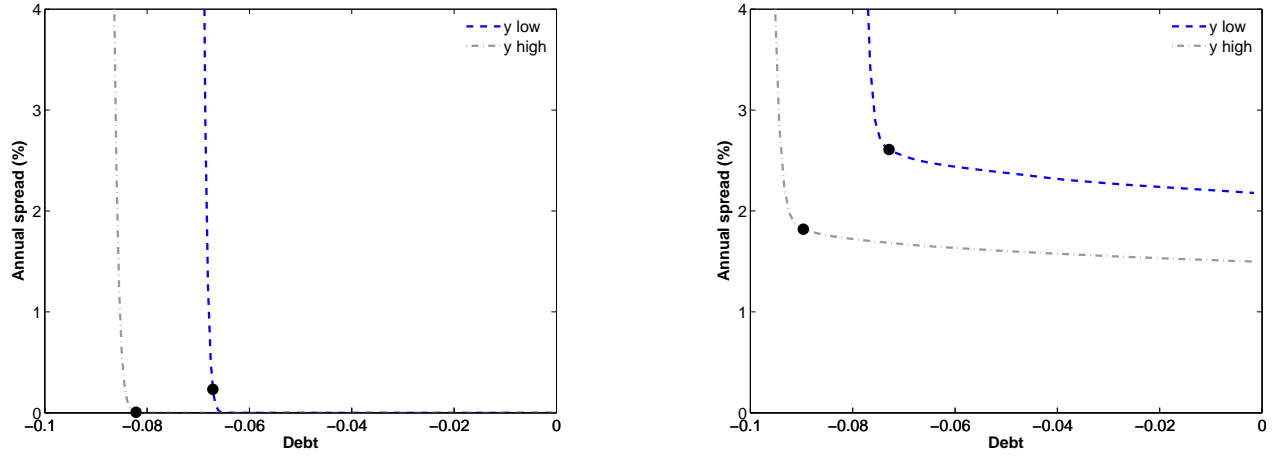


Figure 2: Menu of combinations of spread yields and next-period debt levels ($\frac{b'}{\delta+r}$) available to the government. The left (right) panel is obtained assuming bonds with a duration of one quarter (four years). The solid dots illustrate the optimal decision of a government that inherits a debt level equal to the average debt observed in our simulations for each bond duration. The low (high) value of y correspond to an endowment realization that is two standard deviations below (above) the unconditional mean.

The left-hand side of equation (5) represents the marginal benefit of borrowing. By issuing one extra bond today, the government obtains $q^{ZP}(b', y)$ extra units of current consumption. The right-hand side of equation (5) represents the marginal cost of borrowing. The first term in the right-hand side represents the cost incurred by the government because of the decrease in future consumption implied by current borrowing. The second term in the right-hand side represents the cost incurred by the government because of the decrease in current consumption implied by the fall in the price of current issuances that results from the increase in current borrowing. We shall refer to the second term in the right-hand side of equation (5) as the *current cost of borrowing*.

The current cost of borrowing helps one understand why, both with one-quarter bonds and four-year bonds, the government does not want to choose borrowing levels for which zero-profit spread schedule in Figure 2 is vertical: For these borrowing levels, since the sensitivity of the spread (and the bond price) to the borrowing level is high, the current cost of borrowing is high.

Figure 2 shows that with one-quarter bonds, in order to avoid borrowing levels for which the current cost of borrowing is too high (i.e., the zero-profit spread schedule in is vertical), the

government chooses borrowing levels that imply spreads close to zero (see Aguiar and Gopinath (2006) and Hatchondo et al. (2007) for a more thorough discussion of the government's choice in one-period-bond models). In contrast, with long-duration bonds, the government also stays away from debt levels for which the current cost of borrowing is high, but this occurs at spread levels that are substantially above zero. Recall that the zero-profit spread schedules in Figure 2 describe the borrowing opportunities available to the government. Why do these borrowing opportunities change when the bond duration changes?

In order to answer this question, we should note first that with long-duration bonds, the spread paid by the government does not only depend on the default probability in the next period, but it also depends on the default probability in every other future period. This is described in equation (4), which shows that when $\delta < 1$ (i.e., when bonds promise payments for more than one period), the bond price depends on both the next-period default decision $h(b', y')$ and the price at which bonds could be sold next period, $q^{ZP}(g(h(b', y'), b', y'), y')$. The latter depends on the expected default probability in every other future period. In contrast, with one-period bonds ($\delta = 1$), the bond price only depends on the default probability in the next period and is independent of the default probability in every other future period.

The left panel of Figure 2 shows that with one-quarter bonds, the government can choose next-period debt levels that imply a spread arbitrarily close to zero. This is the case because when the probability of a default in the next period is close to zero, the expected recovery rate—i.e., the fraction of his investment a lender expects to recover—for these debt levels is close to one. These debt levels would not be paid in the next period solely when the economy is hit with extraordinarily low endowment realizations.

In contrast, the right panel of Figure 2 shows that with four-year bonds, even if the government chooses a next-period debt level that is arbitrarily close to zero, the discount in the bond price would reflect a default premium substantially higher than zero. It is still true that when the next-period debt is close to zero, the probability of a default in the next period is negligible. However, equilibrium default probabilities in other future periods are significantly away from zero and, therefore, the expected recovery rate on current issuances is significantly lower than one. Suppose for example that an emerging economy is issuing long-duration debt for the first

time. No matter how small the first issuance is, lenders would anticipate an expected recovery rate lower than one because they would know that the government would continue to issue debt in the future. This implies that the emerging economy does not have the choice to issue small amounts of debts at the risk-free rate.⁹ For any given next-period default probability (including zero), the higher the default probabilities in other future periods, the higher the spread the government pays in the current period.

As one would expect, default probabilities are higher—and, therefore, spreads are higher—when we assume long-duration bonds. The literature on debt dilution discusses how credit risk and a government’s inability to commit to future borrowing levels imply the government has incentives to overborrow and, therefore, to pay higher spreads.¹⁰ An increase in the current borrowing level raises the default probability on all outstanding debt, and thus, it decreases the debt market value—debt dilution occurs. When issuing bonds, the government only internalizes as a cost the dilution of the value of current bond issuances, but it does not internalize as a cost the dilution of the value of bonds issued in past periods. In the model with one-period bonds, when the government decides its current borrowing level, the outstanding debt level is zero, either because the government has honored its debt obligations at the beginning of the period or because it has defaulted on them. Thus, with one-period bonds, the government does not have the option to dilute previously issued debt. The introduction of long-duration bonds incorporates into the model the incentives to overborrow and pay higher spreads discussed in the literature about debt dilution.

Equation (5) shows that the introduction of long-duration bonds reduces the fraction of debt that would need to be rolled over in a given period in order to keep the debt level constant (this fraction is equal to δ). This tends to reduce the current cost of borrowing by decreasing current issuances ($b' - \tilde{b}$ in equation (5)). With one-quarter bonds, the government has to pay back its entire debt stock in every period it does not default, which leads to relatively high issuance

⁹This finding resembles the result in environments where the borrower does not have within-period commitment to exclusive borrowing contracts (see, for example, Bizer and DeMarzo (1992) and Hatchondo and Martinez (2007))

¹⁰See, for instance, Bi (2006), Bizer and DeMarzo (1992), Detragiache (1994), Hatchondo and Martinez (2007), Kletzer (1984), Sachs and Cohen (1982), and Tirole (2002).

volumes. In contrast, with four-year bonds, the government only rolls over a small fraction of its debt, and therefore, the decrease in the issuance price in the current cost of borrowing is weighted by a smaller number (a smaller issuance volume).

It should be mentioned that the low mean spread level in our one-quarter-bond benchmark is a common feature of models of sovereign default. For instance, Aguiar and Gopinath (2006) report spread levels similar to the ones observed in our model with one-period bonds. Benjamin and Wright (2008) also report low spread levels for pre-default periods comparable to the ones used to compute the statistics in Table 2. Mendoza and Yue (2007) are able to replicate the dynamics of sovereign bond spreads before a debt crisis but with spread levels lower than the ones observed in the data. The low spread levels generated by the baseline sovereign default framework motivated Guerrieri et al. (2008) to introduce production into the model. Our results indicate that assuming long-duration bonds instead of one-period bonds could improve the ability of these models to generate spread levels consistent with the ones observed in the data.

We should also mention that Arellano (2008) succeeds in presenting a quantitative model with one-period bonds that generates a significant mean spread. This is achieved by assuming a cost of defaulting that is more sensitive to current income than the one we assume. However, in her simulations, when economic conditions are good, the spread generated by the model is close to zero, which is not observed in the data. The mean spread in Arellano (2008) is close to the mean spread in the data because when economic conditions are bad the spread in her simulations is higher than in the data—see Figure 5 in Arellano (2008). Our findings suggests that even when the default cost resembles the one used in Arellano (2008), introducing long-duration bonds could improve the ability of the model to replicate the spread behavior in the data. The reason is that when economic conditions are good and the next-period probability of default is small, long-duration-bond spreads may be significantly away from zero because lenders anticipate a higher default probability in other future periods.

Next, we discuss why introducing long-duration bonds allows the model to generate a higher spread volatility. Figure 2 shows that in the model with four-year bonds, the menu of interest rates available to the government is more sensitive to the current endowment realization than in the model with one-period bonds. This is the case because lower income levels today imply lower

expected income levels in future periods, and thus, lenders anticipate higher default probabilities in future periods (recall that with long-duration bonds, the interest rate spread depends on default probabilities in every future period, not just in the next period). When four-year bonds are assumed, the higher sensitivity of the menu of interest rates to the endowment leads the government to accept spread levels that are more sensitive to income (see the dots in Figure 2) and, thus, it leads to a higher spread volatility.

We also find that the face value of debt is higher in the model with long-duration bonds compared to the model with one-period bonds. The average face value of debt as a fraction of quarterly output increases from 7.5% in the model with one-period bonds to 8.4% in the model with long-duration bonds.¹¹ There are two reasons for this. First, as explained before, with long-duration bonds, the government is only concerned about the effect of current-period issuances on the price of these issuances, which constitute a small fraction of the total debt level. This induces the government to choose a higher debt burden with long-duration bond. Second, for any given debt burden, the higher default probability obtained with long-duration bonds implies a higher face value of debt. The existence of scenarios in which the government will default implies that the actual debt burden is lower than the face value of future coupon obligations. For a given debt burden, a higher default probability generates a higher face value of debt.

It should also be mentioned that the introduction of long-duration bonds could allow models of sovereign default to match both debt levels and debt service levels observed in the data at the same time. With one-period bonds, debt levels are very close to debt service levels. On the one hand, studies that target debt service levels generate debt levels that are too low compared with the data. On the other hand, studies that target debt levels generate debt service levels that are too high compared with the data. With long-duration bonds, if the bond duration in the model matches the one in the data, and the spread in the model matches the spread in the data, targeting the mean debt level is equivalent to targeting the mean debt service level.

¹¹Debt levels generated by the baseline model of sovereign default are difficult to compare with debt levels in the data. In the model, the government cannot borrow and save at the same time, the recovery rate on debt in default is zero, and all debt is held by foreigners. As it is illustrated in Figure 2, equilibrium debt levels generated by the model are directly linked to the cost of defaulting, which we assume is given by 8.3% of the output level in the default period. The debt level generated by the model could easily be increased by increasing the assumed cost of defaulting.

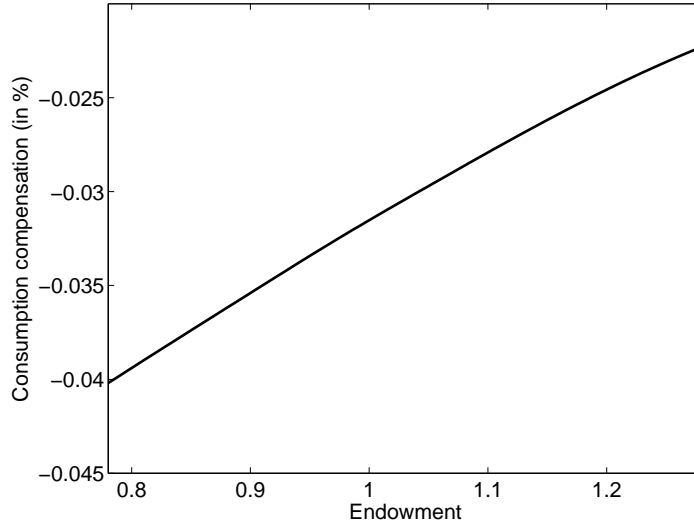


Figure 3: Consumption compensation (in percentage terms) that makes domestic agents indifferent between the economy with one-quarter bonds and the economy with four-year bonds. The figure corresponds to the case where the initial debt level is equal to zero.

Finally, we find that the domestic agents' expected utility is higher in the economy with one-period bonds. Figure 3 illustrates the consumption compensation that would make the domestic agents indifferent between that economy and the economy with four-year bonds. The figure corresponds to cases where the initial debt level is equal to zero. The negative values of the consumption compensation indicate that the expected utility is higher in the economy with one-period bonds. The higher default frequency in the economy with long-duration bonds (which is reflected in higher spreads) implies that the country pays the cost of defaulting more often than in the economy with one-period bonds, which is detrimental to welfare. Of course, there are other tradeoffs between long and short debt that we are not considering in this paper (for a discussion of these tradeoffs see, for example, Alfaro and Kanczuk (2007), Arellano and Ramanarayanan (2006), Bi (2006), and Broner et al. (2007)).

6 Conclusions

We presented an extended version of the baseline model of sovereign default that allows us to study economies where the government issues long-duration bonds. We showed that when the model is parameterized to display a bond duration similar to the one observed in the data, the mean and the volatility of the interest rate are substantially larger than when one-quarter bonds are assumed. Replicating the behavior of interest rates in emerging economies is an important goal in studies that analyze the macroeconomic behavior of these economies. Our results narrow the gap between the predictions of the baseline model and the data, indicating that the extended model is a useful tool for future research.

References

- Aguiar, M. and Gopinath, G. (2006). ‘Defaultable debt, interest rates and the current account’. *Journal of International Economics*, volume 69, 64–83.
- Aguiar, M. and Gopinath, G. (2007). ‘Emerging markets business cycles: the cycle is the trend’. *Journal of Political Economy*, volume 115, no. 1, 69–102.
- Alfaro, L. and Kanczuk, F. (2007). ‘Debt Maturity: Is Long-Term Debt Optimal?’ NBER Working Paper 13119.
- Amador, M. (2003). ‘A Political Economy Model of Sovereign Debt Repayment’. Manuscript, Stanford University.
- Arellano, C. (2008). ‘Default Risk and Income Fluctuations in Emerging Economies’. *American Economic Review*, volume 98(3), 690–712.
- Arellano, C. and Ramanarayanan, A. (2006). ‘Default and the term structure in sovereign bonds’. Working Paper, University of Minnesota.
- Arteta, C. and Hale, G. (2006). ‘Sovereign Debt Crises and Credit to the Private Sector’. Federal Reserve Bank of San Francisco Working Paper 2006-21.
- Athreya, K. (2002). ‘Welfare Implications of the Bankruptcy Reform Act of 1999’. *Journal of Monetary Economics*, volume 49, 1567–1595.
- Athreya, K., Tam, X. S., and Young, E. R. (2007a). ‘Does Rising Income Risk Lead to Better Risk Sharing?’ Manuscript, University of Virginia.
- Athreya, K., Tam, X. S., and Young, E. R. (2007b). ‘A Quantitative Model of Information and Unsecured Lending’. Manuscript, University of Virginia.
- Athreya, K. B. and Janicki, H. P. (2006). ‘Credit exclusion in quantitative models of bankruptcy: does it matter?’ *Economic Quarterly*, volume 92, no. 1, 17–50. Federal Reserve Bank of Richmond.

- Bai, Y. and Zhang, J. (2006). ‘Financial integration and international risk sharing’. Working Paper, Michigan University.
- Benjamin, D. and Wright, M. L. J. (2008). ‘Recovery Before Redemption? A Theory of Delays in Sovereign Debt Renegotiations’. Manuscript.
- Bi, R. (2006). ‘Debt Dilution and the Maturity Structure of Sovereign Bonds’. Manuscript, University of Maryland.
- Bizer, D. S. and DeMarzo, P. (1992). ‘Sequential Banking’. *Journal of Political Economy*, volume 100, 41–61.
- Broner, F. A., Lorenzoni, G., and Schukler, S. L. (2007). ‘Why do emerging economies borrow short term?’ National Bureau of Economic Research Working Paper 13076.
- Chatterjee, S., Corbae, D., Nakajima, M., and Ríos-Rull, J.-V. (2007a). ‘A Quantitative Theory of Unsecured Consumer Credit with Risk of Default’. *Econometrica*, volume 75, 1525–1589.
- Chatterjee, S., Corbae, D., and Ríos-Rull, J.-V. (2007b). ‘Credit Scoring and Competitive Pricing of Default Risk’. Manuscript, University of Texas at Austin.
- Cole, H., Dow, J., and English, W. (1995). ‘Default, settlement, and signaling: lending resumption in a reputational model of sovereign debt’. *International Economic Review*, volume 36, no. 2, 365–385.
- Copeland, T. E. and Weston, J. F. (1992). *Financial theory and corporate policy*. Addison-Wesley Publishing Company.
- Cruces, J. J., Buscaglia, M., and Alonso, J. (2002). ‘The Term Structure of Country Risk and Valuation in Emerging Markets’. Manuscript, Universidad Nacional de La Plata.
- Cuadra, G. and Sapriz, H. (2006). ‘Sovereign default, terms of trade, and interest rates in emerging markets’. Working Paper 2006-01, Banco de Mexico.

- Cuadra, G. and Sapriza, H. (2008). ‘Sovereign default, interest rates and political uncertainty in emerging markets’. *Journal of International Economics*. Forthcoming.
- Cunningham, A., Dixon, L., and Hayes, S. (2001). ‘Analyzing yield spreads on emerging market sovereign bonds’. *Financial Stability Review*, volume December, 175–186.
- D’Erasmus, P. (2008). ‘Government Reputation and Debt Repayment in Emerging Economies’. Manuscript, University of Texas at Austin.
- Detragiache, E. (1994). ‘Sensible buybacks of sovereign debt’. *Journal of Development Economics*, volume 43, 317–333.
- Eaton, J. and Gersovitz, M. (1981). ‘Debt with potential repudiation: theoretical and empirical analysis’. *Review of Economic Studies*, volume 47, 289–309.
- Eichengreen, B. and Portes, R. (2000). ‘Debt Restructuring With and Without the IMF’. Manuscript, UC Berkeley.
- Eyigungor, B. (2006). ‘Sovereign Debt Spreads in a Markov Switching Regime’. Manuscript, UCLA.
- Gelos, G., Sahay, R., and Sandleris, G. (2004). ‘Sovereign Borrowing by Developing Countries: What Determines Market Access?’ IMF Working Paper 04/221.
- Guerrieri, V., Lorenzoni, G., and Perri, F. (2008). ‘International Borrowing, Investment and Default’. Manuscript.
- Hatchondo, J. C. and Martinez, L. (2007). ‘Credit risk without commitment’. Manuscript. Federal Reserve Bank of Richmond.
- Hatchondo, J. C., Martinez, L., and Sapriza, H. (2006). ‘Computing business cycles in emerging economy models’. Working Paper No. 06-11. Federal Reserve Bank of Richmond.

- Hatchondo, J. C., Martinez, L., and Sapriza, H. (2007). ‘Quantitative Models of Sovereign Default and the Threat of Financial Exclusion’. *Economic Quarterly*, volume 93, 251–286. No. 3.
- Hatchondo, J. C., Martinez, L., and Sapriza, H. (2008). ‘Heterogeneous borrowers in quantitative models of sovereign default.’ *International Economic Review*. Forthcoming.
- IMF (2002). ‘Sovereign Debt Restructurings and the Domestic Economy Experience in Four Resent Cases’. Policy development and review department.
- Kaminsky, G. and Reinhart, C. M. (1999). ‘The Twin Crises: The Causes of Banking and Balance of Payments Crises’. *American Economic Review*, volume 89, 473–500.
- Kletzer, K. M. (1984). ‘Asymmetries of Information and LDC Borrowing with Sovereign Risk’. *The Economic Journal*, volume 94, 287–307.
- Kobayashi, K. (2006). ‘Payment uncertainty, the division of labor, and productivity declines in great depressions’. *Review of Economic Dynamics*, volume 9, 715–741.
- Krusell, P. and Smith, A. (2003). ‘Consumption-savings decisions with quasi-geometric discounting’. *Econometrica*, volume 71, no. 1, 365–375.
- Kumhof, M. (2004). ‘Fiscal crisis resolution: taxation versus inflation’. Manuscript, Stanford University.
- Kumhof, M. and Tanner, E. (2005). ‘Government Debt: A Key Role in Financial Intermediation’. IMF Working Paper 05/57.
- Li, W. and Sarte, P.-D. (2006). ‘U.S. consumer bankruptcy choice: The importance of general equilibrium effects’. *Journal of Monetary Economics*, volume 53, 613–631.
- Livshits, I., MacGee, J., and Tertilt, M. (2008). ‘Consumer Bankruptcy: A Fresh Start,’. *American Economic Review*. Forthcoming.

- Lizarazo, S. (2005). ‘Sovereign risk and risk averse international investors’. Working Paper, ITAM.
- Lizarazo, S. (2006). ‘Contagion of financial crises in sovereign debt markets’. Working Paper, ITAM.
- Mendoza, E. and Yue, V. (2007). ‘A Solution to the Default Risk-Business Cycle Disconnect’. Manuscript, New York University.
- Meyersson, E. (2006). ‘Debt intolerance and institutions, an empirical investigation into the institutional effect of sovereign debt’.
- Neumeyer, P. and Perri, F. (2005). ‘Business cycles in emerging economies: the role of interest rates’. *Journal of Monetary Economics*, volume 52, 345–380.
- Sachs, J. and Cohen, D. (1982). ‘LDC borrowing with default risk’. National Bureau of Economic Research Working Paper 925.
- Sánchez, J. M. (2008). ‘The Role of Information in Consumer Debt and Bankruptcy’. Manuscript, University of Rochester.
- Sandleris, G. (2006). ‘Sovereign Defaults: Information, Investment and Credit’. Manuscript, Johns Hopkins.
- Schmitt-Grohé, S. and Uribe, M. (2003). ‘Closing small open economy models’. *Journal of International Economics*, volume 61, 163–185.
- Tirole, J. (2002). ‘Financial Crises, Liquidity, and the International Monetary System’. Princeton University Press.
- Uribe, M. and Yue, V. (2006). ‘Country spreads and emerging countries: Who drives whom?’ *Journal of International Economics*, volume 69, 6–36.
- Wright, M. L. J. (2002). ‘Reputations and Sovereign Debt’. Manuscript, Stanford University.

Wright, M. L. J. (2005). ‘Coordinating Creditors’. *The American Economic Review, Papers and Proceedings*, volume 95, no. 2, 388–392.

Yue, V. (2005). ‘Sovereign default and debt renegotiation’. Working Paper, NYU.