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Abstract

Many emerging economies maintain significant positions in both external sovereign debt and foreign reserves, paying spreads of over 250 basis points on average. Arguments advanced in empirical work and policy discussions suggest that governments may do this because international reserves play a role in reducing the likelihood of sovereign debt crises, improving a country's access to debt markets. This paper proposes a model that justifies that argument. The government makes optimal choices of debt and reserves in an environment in which self-fulfilling rollover crises *a-là* Cole-Kehoe and external default *a-là* Eaton-Gersovitz coexist. This allows for both fundamental and market-sentiment-driven debt crises. Self-fulfilling crises arise because of a lender's coordination problem when multiple equilibria are feasible. Conditional on the country's Net Foreign Asset position, additional reserves make the sovereign more willing to service its debt even when no new borrowing is possible, which enlarges the set of states in which repayment is the dominant strategy and, hence, reduces the set of states that admit a self-fulfilling crisis. From an ex-ante perspective, reserves reduce the probability of crises *in the future* which lowers *current* sovereign spreads. Quantitatively the model can explain 50% of Mexico's international reserves holdings, while accounting for key cyclical facts.

1 Introduction

Several emerging economies have both foreign currency denominated debt outstanding and international reserve holdings. This coexistence is costly for the government, since it pays a higher interest rate on its debt than what it earns on its reserves holdings. This suggests there should be some extra *benefit* of reserve holdings.

The right panel of Table 1 presents the foreign debt stock and reserves holding as a percentage of GDP in 2015 for a panel of emerging economies. The median country held 16.9% of GDP in reserves while owing 14.9% in foreign currency and paying a spread of 238 basis points over the return on reserves. The numbers from 2015 are not only a product of the recent trend in emerging economies reserves accumulation. The left panel on Table 1 shows the 15 year average, between 2001 and 2015, of debt, reserves and sovereign spreads for the same countries. The data shows that the concurrence of debt and reserves is not a short-term phenomenon.

Table 1: External Debt, Reserves and Spreads

Country	Averages (2001-2015)			Latest (2015)		
	Reserves [1]	Debt [1]	Spread [2]	Reserves[1]	Debt[1]	Spread [2]
Argentina	9.8	31.3	1478	6.0	18.7	458
Brazil	11.5	7.0	391	23.2	4.4	446
Bulgaria	31.7	18.1	196	45.5	11.9	68
Chile	15.2	2.9	147	16.8	2.8	226
Colombia	10.7	13.1	298	18.3	16.6	332
Indonesia	12.0	21.0	282	12.1	17.0	317
Lithuania	15.0	20.9	228	6.5	31.1	122
Mexico	10.1	12.0	187	16.3	18.7	216
Peru	24.3	21.2	268	32.9	11.1	235
Philippines	23.4	24.0	298	28.3	10.8	121
Poland	16.5	26.6	73	20.1	29.4	51
South Africa	10.1	9.2	214	16.8	16.7	406
Turkey	12.0	13.3	356	16.1	13.1	322
Ukraine	16.0	15.3	712	6.6	30.3	1621
Median	15.1	14.3	248	16.9	14.9	238

The main reason cited for emerging markets reserve holdings is known as the precautionary motive: reserves are a liquidity buffer that protects the sovereign against adverse developments in financial markets. This motive for holding reserves has been adopted by the mainstream policy institutions, as the following excerpt from the reserves definition in the 6th edition of the IMF's Balance of Payments Manual shows:

"external assets . . . controlled by monetary authorities for meeting balance of payments financing needs, for intervention in exchange markets . . . , and for other related purposes (such as maintaining confidence in the currency and the economy, and serving as a basis for foreign borrowing)."

Other explanations, like intergenerational wealth transfers and exchange rate management, have been suggested to account for the holding of reserves, but they are still problematic. Although some of those countries with high international reserves have positive Net Foreign Asset (NFA) positions -like China-, the majority of emerging economies are still indebted vis-a-vis the rest of the world, which precludes the wealth transfer motive. Similarly, several of those countries have floating exchange rate regimes, weakening the case for an export promotion or currency management motive.

This paper focuses on the precautionary motive for holding reserves, in particular, in their role in preventing debt crises, maintaining confidence in the economy, and enhancing foreign borrowing conditions. Empirical evidence supports this case: Ben-Bassat and Gottlieb (1992), Calvo, Izquierdo, and Loo-Kung (2013) and Tavares (2015) have found that reserves are correlated with smaller spreads and a lower probability of sudden stops, defined as large current account reversals.

Figure 1: Scatter diagram of spreads and reserves
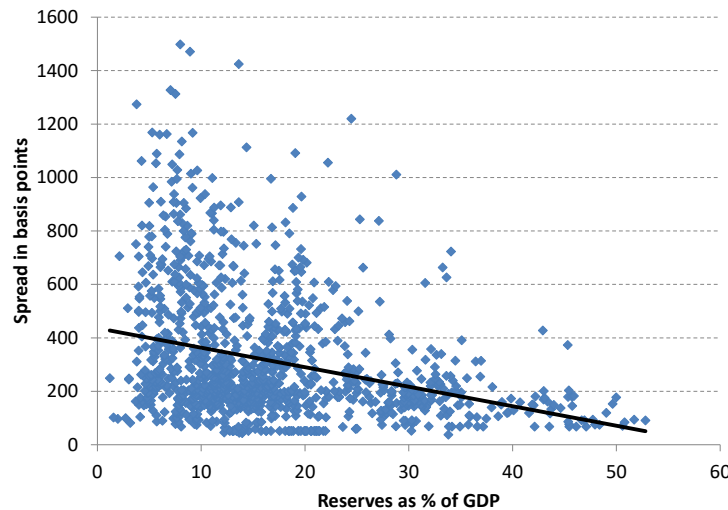
Each dot represents a country-quarter pair for each economy in the sample and quarter between 1994-Q1 and 2015-Q4. A linear relationship between the reserves to GDP ratio and the EMBI spread (in basis points) was added. Additional information about the data and sample used can be found in the data appendix C.

Figure 1 hints at the effect of reserves in reducing spreads. The scatter plot shows reserves-to-GDP ratio and sovereign spreads where each point represents a country and

quarter in the dataset, described in Appendix C. A simple linear relation fitted between the two variables shows a negative correlation.

Ben-Bassat and Gottlieb (1992), Tavares (2015), Gumus (2016), and Levy Yeyati (2008) among others, [1] build country panels and estimate the effect of reserves on the sovereign spreads under different specifications. They find estimates for the coefficient of the reserves-to-GDP ratio on sovereign in the range [-8.0, -2.5] which indicates that for each percentage point increase of the reserves-output ratio the sovereign spread falls between 2.5 and 8 basis points. Table 2 presents estimates of a common panel data regression specification regressing spreads on lagged reserves, debt, current account, output growth, an exchange rate regime dummy and the aggregate EMBI+ spread. The results are in line with previous studies, in particular, the point estimate of the spread reduction, after a 1% increase in the reserves-to-GDP ratio, is between 5.5 and 7.6 basis points.

In a similar setting, Calvo et al. (2013) use a panel of emerging and developed countries to assess the impact of international reserves in the cost and probability of sudden-stops, and find that reserves holdings reduce both of them. [2] Gourinchas and Obstfeld (2012) find that international reserve holdings significantly reduce the probability of future crisis (default, currency and banking) on emerging economies, in particular they report a marginal effect of the reserves-to-GDP ratio on the default probability of -0.59.

On the theoretical side, however, sovereign debt models with strategic default struggle to incorporate reserves. International reserves are protected by the sovereign immunity clause, which means they are not pledgable and cannot be seized by creditors in case of a default. This makes more challenging for these models to sustain both debt and reserve holdings since reserves make default more attractive. Bulow and Rogoff (1989) showed that if the sovereign is allowed to accumulate assets after default, reputation costs are not enough to support lending since default happens when debt is at its highest. With reserves, a front-loaded version of this mechanism applies, since the sovereign can increase its debt quickly and default on it in the next period, keeping the proceeds from debt issuance as reserves.

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[1] See Petrova, Papaioannou, and Bellas (2010) for a survey of papers assessing the effect of reserves and other fundamental variables on sovereign spreads. For a survey of the literature on reserves and the probability of crises see Bussière, Cheng, Chinn, and Lisack (2015).

[2] Calvo et al. (2013) assume that the probability of sudden stops comes from a Probit model including reserves, the output cost of default is linear in reserves, and there is a constant cost of holding reserves. The first order condition for reserves is obtained analytically. Then they proceed to estimate the Probit model for the likelihood of sudden-stops and the linear model for the output cost, and include those estimates in their first order condition. Given the specification uncertainty, they assume the sovereign chooses the most conservative model, which is the one yielding larger reserves. For Mexico, they find the optimal reserve to GDP ratio to be 15% in 2007 and 22% in 2010.

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Table 2: Effect of reserves on spreads

	Pooled OLS	Random Effects	Fixed Effects
L. Reserves/GDP	-7.559***	-5.512***	-5.561**
	(1.141)	(2.032)	(2.260)
L. Debt/GDP	2.964**	6.911***	7.797***
	(1.226)	(1.930)	(2.059)
L. Agg. EMBI+	0.369***	0.324***	0.313***
	(0.080)	(0.069)	(0.069)
L. Current Account/GDP	-3.807*	-10.397***	-10.907***
	(2.095)	(2.045)	(2.072)
L. Exchange rate regime	205.070***	136.424***	124.703***
	(23.888)	(34.515)	(36.439)
L.GDP growth	-71.471***	-62.485***	-62.371***
	(7.186)	(5.968)	(5.987)
Cons.	186.433***	131.684*	152.143**
	(40.297)	(70.990)	(62.884)
R^2	0.313	0.280	0.304
N	650	650	650

Notes: EMBI spread (basis points) as dependent variable. All ratios are calculated using the current GDP; the Reserves to GDP ratio and Debt to GDP is calculated as the variable divided by four times the quarterly GDP (annualized). Exchange rate regime corresponds to a dummy variable that equals 1 for countries with a fixed exchange rate regime. Additional information about the data used can be found in the appendix. Standard errors in parentheses. The R^2 statistic of the random effects model (column 2) corresponds to the overall R^2. * Significance at the 10 percent level. ** Significance at the 5 percent level. *** Significance at the 1 percent level.

Recent developments in quantitative models of sovereign debt and international reserves have had some success in explaining the coexistence of debt and reserves by stressing the hedging properties of international reserves in the event of a sudden stop. Jeanne and Rancière (2011) highlight the insurance role of reserves against rollover crises in a simple model with switching exogenous collateral constraints. With those insights in mind, Bianchi, Hatchondo, and Martinez (2012) added exogenous rollover crises to an Eaton-Gersovitz style model of long-term sovereign debt. Their model can fully explain reserves accumulation in Mexico while keeping track of standard cyclical moments, but the probability of those exogenous crises has to be introduced as a function of reserves.[3]

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[3]In their latest version Bianchi, Hatchondo, and Martinez (2016) drop the exogenous rollover crises and instead use a risk aversion shift on the lenders' side, exogenously tied to the sovereign's income shock. They mention that this link accounts for half of the reserves holdings explained by their numerical exercise.

While these papers are able to represent the empirical facts, they model the link between the probability of sudden stops and the reserves as an exogenous object, which does not allow the authors to assess if those holdings are useful for preventing sovereign debt crises. This contrasts sharply with the findings of empirical studies suggesting that international reserves help to prevent debt crises. Those empirical regularities have been used to build several measures of international reserves adequacy, assess optimal reserve holdings, and even prescribe policy, but there is little understanding of the mechanism that drives them.

In the model developed in this paper, self-fulfilling debt crises as developed by Cole and Kehoe (2000) are introduced. Reserves affect the probability of a sudden stop generated from a self-fulfilling debt crisis via an endogenous mechanism. In choosing the share of international reserves in the sovereign's portfolio, the government takes into account that the frequency of those crises is determined by the extent to which reserves can move the economy in and out of states of nature in which the coordination problem may give way to a self-fulfilling debt crisis. This provides an explanation for reserves accumulation that does not depend on *ad-hoc* assumptions about their effect on the dynamics of the sovereign's economy or on other debt market participants.

Figure 2: Countries with spread 2 s.d. above mean
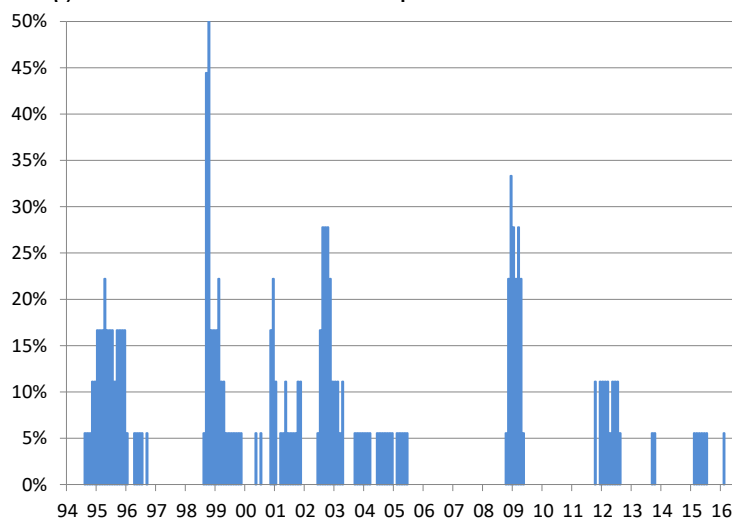
This figure shows the percentage of countries from the sample whose EMBI spread is more than two times the standard deviation above its corresponding mean. Additional information about the data and sample used can be found in the data appendix.

One fact that is indicative of the potential empirical relevance of self-fulfilling debt crises is the observation of several periods in which countries with different fundamentals all face spread widening and current account reversals. Figure 2 shows the percen-

tage of countries with spread two standard deviations above their corresponding means in each month over the 1994-2015 period. By construction, when multiple countries are facing an unusually high spread, the aggregate spread must be higher as well. Calvo, Izquierdo, and Mejía (2008) identified and named these events Systemic Sudden Stops (3S): periods with high aggregate spreads and where multiple countries face simultaneous current account reversals. These 3S periods suggest a driving external financial factor. Other recent studies have presented evidence in the same line: Aguiar, Chatterjee, Cole, and Stangebye (2016b) found a common exogenous factor driving spreads for a panel of emerging markets. Tavares (2015) includes a time effect in his empirical specifications for the sovereign spread. In Table 2, the coefficient on the EMBI+ spread, which is the same across all countries, captures this effect.

This paper contributes to the existing literature by setting up a model that not only can quantify the impact of international reserves on the probability of self-fulfilling debt crises but also produces an endogenous mechanism linking both. In the proposed environment self-fulfilling crises arise as a coordination failure in the financial markets but *only when fundamentals allow for doubts about repayment by the sovereign*.

The intuition for the mechanism is that, conditional on a Net Foreign Asset position (NFA), additional reserves make the sovereign more willing to service its debt even if there is an episode of market panic and no new borrowing is possible. This reduces the set of states that admit a self-fulfilling crises. Seen from an *ex-ante* perspective, the additional holdings of international reserves reduce the probability of a self-fulfilling debt crisis in the future which in turn decrease current sovereign spreads.

The quantitative implications of the model are studied in a numerical exercise calibrated to Mexico. The model can account for 50% of the country's international reserve holdings, while matching the average outstanding debt, the volatility of the sovereign spread and Mexico's default frequency. The model also features the negative correlation between spreads and reserves observed in the data.

The model can be used to quantify the optimal portfolio of reserves and external debt for the sovereign. Additionally, this framework can explain empirical regularities regarding reserves holdings and measures of crises vulnerability used in policy circles. The Guidotti-Greenspan rule, which links the probability of sudden stops to the ratio of reserves to short term debt, is the most popular reserve adequacy metric among policymakers but not the only one. [4] This paper proposes a micro-founded explanation for

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[4]Other measures include ratios of reserves to several variables like: monthly imports, current account deficit, total debt and short term debt. In addition, according to the IMF's survey of reserve managers, countries approach to assess reserve adequacy usually include a mixture of these ratios.

that and other empirical regularities, by endogenously tying the probability of crises to economic fundamentals. Finally the setup is able to assess the effectiveness of contingent lending policies put in place by the IMF and developed countries' central banks aimed at preventing *confidence* crises. [5]

1.1 Related literature

This paper relates to the sudden stops literature which has been able to explain reserve holdings as *self-insurance* against the risk of sudden reversal in financing. Jeanne and Rancière (2011) highlight the insurance role of reserves against sudden stops in a simple model in which an exogenous collateral constraint suddenly switches preventing new borrowing. Durdu, Mendoza, and Terrones (2009) present a model in which a planner internalizes how the choice of reserves affects the probability and magnitude of sudden stops driven by the deflation mechanism. [6] However, these papers abstract from modeling sovereign default and the behavior of lenders while this paper models sovereign default and the response of lenders to reserves accumulation, which is key for the role of reserves in preventing crises.

This paper builds on the long strain of strategic sovereign default models pioneered by Eaton and Gersovitz (1981) and further developed by Aguiar and Gopinath (2006) and Arellano (2008) whose work highlighted the quantitative relevance of that setup by replicating key business cycle statistics for emerging market economies. [7] In these models however, the sovereign was not allowed to simultaneously hold assets and debt.

The first attempt to model both debt and reserves in a setup with default was proposed by Alfaro and Kanczuk (2009). They found that reserves are too costly to hold in equilibrium. With only one-period debt, the net position (reserves minus debt) is the relevant state after the sovereign chooses to repay. But after default, reserves can com-

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[5]IMF's Flexible Credit Line(FCL) is one of those contingent credit facilities. This was aimed at countries with strong fundamentals for crisis-prevention and crisis-mitigation lending. It allows the sovereign to draw on the credit line at any time (within a pre-specified period) which turned those funds into contingent reserves without imposing additional conditions for disbursement. Three countries: Colombia, Mexico and Poland used the FCL but none of them withdrew any funds. However it is still believed that at the peak of the global financial crisis in 2009 the mere existence of this arrangement injected confidence in the markets. Given our endogenous self-fulfilling crises framework, our model is in a better position to address the impact of this instrument in precluding the possibility of such events. Bocola and Dovis (2015) use a similar framework to evaluate the ECB's promise to buy European sovereign bonds (Draghi's *"whatever it takes"*) on European spreads.

[6]Durdu et al. (2009) do not model the portfolio choice of debt and reserves, the latter arise as a higher NFA position. Also their model is not of sovereign default but one of private debt crises driven by Fisherian deflation.

[7]A comprehensive review of the literature that followed can be found in Aguiar et al. (2016b).

pensate the direct output costs and allow for consumption smoothing, making exclusion from financial markets more bearable. Hence, conditional on a NFA, more reserves make the sovereign less likely to repay. Lenders react to this by increasing spreads and in equilibrium debt stocks fall. In this paper, reserves still make default more attractive for the sovereign, but allowing for long-term debt and self-fulfilling crises makes the gross portfolio position relevant after repayment, allowing a role for reserves.

More recently, Bianchi et al. (2012) set up a model of endogenous default, long-term debt and exogenous roll-over crises, which can explain Mexico's reserves holdings . In their model the probability of roll-over crises is assumed to be a decreasing function of reserves. In contrast, the framework developed here endogenously ties the probability of crises to the fundamentals of the economy, and in particular to reserves holdings, which allows for them to play a role in crisis prevention.

This paper also draws from the self-fulfilling debt crises literature which has two main branches: the first one pioneered by Calvo (1988), features multiple current debt prices consistent with future consumption and default decisions. Recent papers on the Calvo tradition are Lorenzoni and Werning (2013) and Nicolini, Teles, Ayres, and Navarro (2015). In a setup similar to bank run, Cole and Kehoe (2000) develop a framework in which both repayment and default can be equilibrium actions depending on the lenders' decision to roll over debt or not. Recently Chatterjee and Eyigungor (2012) introduce this setup in their Eaton-Gersovitz model of long term debt to make its issuance a superior alternative to short-term bonds.

The self-fulfilling debt crises environment has also been used to assess the role of an International Lender Of Last Resort (ILOR): Roch and Uhlig (2016) introduce an ILOR and determine the minimal intervention needed to rule out self-fulfilling crises. Bocola and Dovis (2015) measure the role of self-fulfilling crises in recent European debt crises and the impact of the ECB's Outright Monetary Transactions (OMT) announcement. This paper complements that literature, as reserves emerge as an alternative to the insurance and crisis prevention role of the ILOR.

Other attempts have given reserves different roles to explain their coexistence with debt. Tavares (2015) gathers empirical evidence that confirms reserves reduce sovereign spreads, then proposes a model in which reserves are necessary after a default to make a settlement on a fraction of the stock of defaulted debt. Building on the Diamond and Dybvig (1983) bank run framework, Hur and Kondo (2013) set up an optimal contracting problem to find a role for reserves as a collateral that prevents runs on the sovereign debt. In this paper reserves are not given any additional role besides being a non-seizable cash buffer, which makes the task of explaining their coexistence more challenging.

This paper is also related to the credit card puzzle literature of Telyukova (2013) and Telyukova and Wright (2008). In these models, the existence of certain goods that cannot be purchased with credit cards can explain the coexistence of cash balances and credit card debt in households portfolios. This is similar to the sudden stop literature in which the reserves balances of the sovereign can be explained by the existence of certain states where credit is not available. However the legal environments for the two problems differ substantially. Bankruptcy laws prevent households and corporations from holding assets after a default event while international reserves are protected by the Sovereign Immunity clause and cannot be seized by lenders. Hence, cash makes the household credit safer while reserves make the sovereign riskier.

The rest of the paper continues as follows: Section 2 presents the empirical facts. In Section 3 the model is specified and its theoretical properties are discussed. Section 4 presents the quantitative analysis and results. Finally, Section 5 concludes and states directions of further research.

2 The Model

The model is based on the classic sovereign default model proposed by Eaton and Gersovitz (1981) with the modifications to introduce long-term debt and self-fulfilling debt crises incorporated by Chatterjee and Eyigungor (2012). The main addition is the introduction of foreign reserves (an asset that the sovereign can hold) and the portfolio choice of debt and reserves.

2.1 The Sovereign

2.1.1 Preferences, endowments, choices

The sovereign seeks to maximize the representative agent's utility:

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right],$$

where $u(\cdot)$ is a twice-continuously differentiable utility function, c_t is consumption and $\beta < 1$ is the discount factor.

The model can be thought of as a benevolent government maximizing private utility subject to resource constraints, or as the government's own utility considering its own expenditures and revenues. As is common in the literature, the former is assumed here.

The resource constraint faced by the government varies depending on whether it chooses to repay or not, and on whether there is a self-fulfilling debt crises preventing it from accessing credit markets. Below, the resource constraints under each scenario and the corresponding recursive maximization problem are described.

In all of these scenarios, the government draws a realization of endowment income process denoted y_t. The log-income ($\log(y_t)$) follows an AR(1) process.

The sovereign has access to two financial instruments. First, an asset, denoted a_t, that represents foreign reserve holdings and pays an exogenous world-determined real rate of return r. Second, long-term bonds specified as in Chatterjee and Eyigungor (2012).

In particular, the government issues long-term bonds described by two parameters: their coupon z and their maturity $\frac{1}{\lambda}$. Only one fixed type of (z, λ) bond can be issued. Each period a fraction $\lambda \in (0, 1)$ of the bond matures. The remaining $(1 - \lambda)$ pays the coupon z and is automatically rolled over. By keeping (z, λ) fixed, the model only needs to keep track of current outstanding debt. Remaining bonds look exactly like newly issued bonds. Hence the law of motion of the outstanding debt law of motion is: [8]

$$b' = (1 - \lambda)b + current.issuance$$

The government has the option to default on previous obligations and default entails two different costs: first, the government loses access to credit markets, with the standard exogenous probability of re-entry $\epsilon \in (0, 1)$; second, the government loses a fraction of the income $\phi(y_t)$ while it remains excluded from credit markets. As Arellano (2008) shows, this is necessary for the model to be able to generate debt stocks consistent with the observed levels and to make default occur at low income realizations. When the government defaults, it is immediately excluded from borrowing in the same period the decision is made. Upon re-entry, all previous obligations are void.

2.1.2 The Government's Problem

The timeline of decisions is as follows: the government enters the period with reserve holdings a and a stock of outstanding debt b if it was not in default state the previous period. First the exogenous shocks are realized: income y, the sunspot variable ω and the reentry shock if the government was in default. After that, if the government has

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[8]Hatchondo and Martinez (2009) and Arellano and Ramanarayanan (2012) also incorporate long term bonds in similar environments but with bonds limited to be a geometrically declining series of coupons, which is the special case of the setup developed by Chatterjee and Eyigungor (2012) when $z = 1$. Here z is set to be equal to the spread plus the risk-free rate, such that on average bonds are issued at par, as is common practice, and nominal and face values are roughly the same which facilitates comparisons.

access to financial markets the state of the economy is $s = (y, a, b, w)$ and the value for the government as a function of the state is denoted $W(y, a, b, w)$. The government then chooses whether to repay ($\delta = 0$) or default ($\delta = 1$), and the portfolio for next period (a', b'). *Simultaneously* the lenders decide whether to enter the bond auction or not. When lenders enter the auction and the government repays, the value for the government is $V^+(y, a, b, w)$; if the government repays but lenders do not enter the auction, the value for the government is $V^-(y, a, b, w)$; and when the government defaults, its debt obligations are void and the value is $X(y, a, w)$, which is the same value achieved by a government that was in default state the previous period and did not gain access to the market in the current period.

When the sovereign chooses to repay in the current period, it has access to the international capital markets and hence it can choose next period debt b', reserves a' and current consumption c taking as given current income y, reserve holdings a and debt outstanding b. It faces a pricing schedule for debt $q(y, a', b', w)$ that depends on future debt and reserves, and on current income y and a sunspot variable $w \in \{0, 1\}$. When $w = 1$ the best equilibrium price schedule is offered by the lenders, and when $w = 0$ they coordinate on the worst possible pricing schedule and a self-fulfilling crisis may arise (see section 2.2). This sunspot variable follows a two-state Markov process to mimic the tranquil and panic regimes in the global financial markets.

Let $V^+(y, a, b, w)$ be the value function corresponding to the sovereign problem in case it has access to international capital markets, then:

$$V^+(y, a, b, w) = \max_{a', b', c} \left\{ u(c) + \mathbb{E}_{y, w} \left[\beta W(y', a', b', w') \right] \right\},$$

$$\text{s.t.} \quad c = y + a - \frac{a'}{1+r} - \left[\lambda + (1-\lambda)z \right] b + q(y, a', b', w) \left[b' - (1-\lambda)b \right].$$

(1)

Under repayment, consumption equals the income realization plus the net resources generated by reserves and debt, which are equal to the economy's balance of trade. The resources generated by reserves are given by $a - \frac{a'}{1+r}$, where a is the payout on reserves carried over from the previous period and $\frac{a'}{1+r}$ is the resource cost of buying reserves a' at the price $\frac{1}{1+r}$. The term $\left[\lambda + (1-\lambda)z \right] b$ captures the debt service cost, which includes the fraction of outstanding debt maturing λb and the coupon payment on the non-maturing part $z(1-\lambda)b$. Finally, current issuance of new bonds is $\left[b' - (1-\lambda)b \right]$ and the revenue collected is $q(y, a', b', w) \left[b' - (1-\lambda)b \right]$.

Note bond prices are forward looking but because of the persistence of the stochastic processes of y and w, their current realizations provide information that is relevant for

forecasting their future realizations, which is in turn relevant for expectations of future utility and prices.

When the government has chosen default and is outside the international markets it takes as given its income, stock of reserves and sunspot variable, $s = (y, a, w)$. In this case ω has no contemporaneous effect on consumption and there is no debt issuance, but it helps to predict ω' which matters for the continuation values W and X next period. Let the value function in this situation be $X(y, a, \omega)$:

$$
\begin{aligned}
X(y, a, \omega) = \max_{c, a'} \Big\{ & u(c) + \beta(1 - \epsilon) \, \mathbb{E}_{y, \omega} \left[X(y', a', \omega') \right] \\
& + \beta \epsilon \, \mathbb{E}_{y, \omega} \left[W(y', a', 0, \omega') \right] \Big\},
\end{aligned}
\tag{2}
$$

$$
\text{s.t} \quad c = y - \phi(y) + a - (1 + r)^{-1} a'.
$$

Under default, consumption equals the income realization plus the net resources generated by reserves. Income in this case is $y - \phi(y)$ where $\phi(\cdot)$ is the direct income cost of default. The resources generated by reserves are given by $a - \frac{a'}{1+r}$, where a is the payout on reserves carried over from the previous period and $\frac{a'}{1+r}$ is the resource cost of buying reserves a' at the price $\frac{1}{1+r}$.

The value function V^- represents the value the sovereign obtains in states in which it chooses to repay but it cannot issue new debt because lenders refused to enter the auction. It is very similar to the function V^+ but with an extra constraint forbidding new bond issuance $b' \leqslant (1 - \lambda)b$.

$$
V^-(y, a, b, \omega) = \max_{a', b'} \Big\{ u(c) + \mathbb{E}_{y, \omega} \left[\beta W(y', a', b', \omega') \right] \Big\},
$$

$$
\text{s.t.} \quad c = y + a - \frac{a'}{1 + r} - \left[\lambda + (1 - \lambda)z \right] b + q(y, a', b', \omega) \left[b' - (1 - \lambda)b \right].
\tag{3}
$$

$$
b' \leqslant b(1 - \lambda).
$$

To complete the formulation of the sovereign's problem, it is needed to define the unconditional value function W which is determined by the default decision.

Given the equilibrium in the default rollover game (defined in the next section), the

sovereign's default decision is:

$$\delta^*(y, a, b, \omega) = \begin{cases} 0 & \text{if } X(y, a, \omega) \leqslant V^-(y, a, b, \omega) \\ 1 & \text{if } X(y, a, \omega) > V^+(y, a, b, \omega) \\ 0 & \text{o/w if } V^-(y, a, b, \omega) < X(y, a, \omega) \leqslant V^+(y, a, b, \omega), \omega = 1 \\ 1 & \text{o/w if }, \omega = 0, \end{cases} \tag{4}$$

where $\delta(\cdot) = 1$ indicates default and $\delta(\cdot) = 0$ repayment. The unconditional value ($W(\cdot)$) of the sovereign before the default decision is made, implied by the default decision above is:

$$W(y, a, b, \omega) = \begin{cases} V^+(y, a, b, \omega) & \text{if } X(y, a, \omega) \leqslant V^-(y, a, b, \omega) \\ X(y, a, \omega) & \text{if } X(y, a, \omega) > V^+(y, a, b, \omega) \\ V^+(y, a, b, \omega) & \text{o/w if }, \omega = 1 \\ X(y, a, \omega) & \text{o/w if }, \omega = 0 \end{cases} \tag{5}$$

This function encompasses the default decision $\delta(\cdot)$. Whenever $W(\cdot) = X(\cdot)$ the sovereign is optimally choosing to default $\delta^*(\cdot) = 1$, and it is choosing to repay in the other cases. In the first case $X \leqslant V^- \leqslant V^+$, repayment is better no matter if there is new lending or not. In the second $X > V^+ \leqslant V^-$ and default is the dominant strategy. But when $V^- < X < V^+$ multiplicity arises and the choice depends on ω. The next section presents the auction game and the reasoning behind this value function and the default-repay decision.

2.2 Rollover crises

Self-fulfilling crises are introduced as in Chatterjee and Eyigungor (2012). In every period, the sovereign and the lenders make simultaneous decisions. The sovereign chooses between default and repaying and the lenders choose whether to enter the auction for newly issued bonds or not. Multiplicity will arise when the government finds it better to repay if it can issue new bonds, but rather defaults if there is no new lending.[9]

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[9]Recent work by Auclert and Rognlie (2016) established uniqueness of equilibrium in the Eaton-Gersovitz type of models under very mild conditions. However, they acknowledge that multiplicity exists when long-term debt is present (as in Chatterjee and Eyigungor (2012)) and when the sovereign savings are unbounded, both of which are true in this setup.

2.2.1 The Game of Default and Rollover

The government and the lenders will play a default-rollover game every period.

The government has two actions, default or repay. The payoffs for the sovereign come from the value functions defined in the previous section. If the government defaults, it earns $X(y, a, \omega)$ no matter what the lenders do. This assumes that in the case of a default, the sovereign does not receive the funds set aside by the lenders that entered the auction. When the government repays and all lenders enter the auction, the sovereign gets $V^+(y, a, b, \omega)$, but if instead no lender enters the auction, the sovereign obtains $V^-(y, a, b, \omega)$.

There is a continuum of risk-neutral and symmetric lenders each endowed with a finite amount of funds. New lenders are always available each period. Previous bond-holders cannot limit the actions of new lenders unless the country is excluded from the financial markets after a default event. Lenders have two possible actions, enter the bond auction and lend, or stay out and not lend. Lenders form beliefs on what other lenders and the government are doing and at equilibrium those beliefs are consistent with the actions of the players. When a lender choose not to lend she will earn zero no matter what other players do. When all lenders choose to lend and the sovereign chooses to repay, the lenders drive the bond price down until they break even, earning zero. If a lender enters the auction and the sovereign defaults, the lender loses an amount $\Delta > 0$ for the opportunity cost. [10]

All players move simultaneously. The equilibrium concept here is a pure-strategies Markov equilibrium. [11]

When the combined funds of the set of lenders that entered the auction is less than the breakeven price for the bond times the new issuance, the bonds are sold at a lower price.

In equilibrium, symmetry implies all lenders choose the same action unless both actions yield the lenders the same utility. But for the lenders to be indifferent, it must be the case that the sovereign is repaying (because $\Delta > 0$) and that the bond price in the auction is the break-even price (a lot of lenders enter the auction). This implies that either the auction is fully subscribed or no lender enters, and the outcome is equivalent

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[10]The nature of this cost Δ is immaterial for the equilibrium as long as it is positive. It can be equal to the total bid the lender made, or just a small fraction representing as the opportunity cost of setting aside funds for an auction.

[11]Recently Aguiar et al. (2016b) generalized this timing structure, allowing for a detailed within period auction-settlement separation and microfounding some of the assumptions made here. In the end they showed that under pure strategies the equilibrium is exactly the same as the one coming out from this setup. In that sense this setup is a simplified version of theirs.

to the one under the assumption that all lenders do the same. Hence, the payoff matrix is given by:

	Repay	Default
Lend	$0, V^+(s)$	$-\Delta, X(s)$
Don't Lend	$0, V^-(s)$	$0, X(s)$

Conditional on a state vector $s = (y, a, b, \omega)$ the game reduces to a two player simultaneous-move game described by the payoff matrix above. Its Nash Equilibria depend on the government's payoffs.

Case 1) If $X(s) < V^-(s) < V^+(s)$, then repay is the *strictly dominant* strategy for the sovereign and $(\text{lend}, \text{repay})$ is the unique Nash equilibrium.

Case 2) When $V^-(s) < V^(s)+ < X(s)$ default is the strictly dominant strategy and thus $(\text{Don}'\text{t}, \text{Default})$ is the only Nash equilibrium.

Case 3) If $V^-(s) < X(s) < V^+(s)$ then there is no dominant strategy. In this case both $(\text{lend}, \text{repay})$ and $(\text{Don}'\text{t}, \text{Default})$ are Nash Equilibria. Here the sunspot variable $\omega \in \{0, 1\}$ plays a role: when $\omega = 1$ all agents coordinate to the $(\text{lend}, \text{repay})$ equilibrium, otherwise the $(\text{Don}'\text{t}, \text{Default})$ is played.

2.2.2 Lenders' optimality condition

Let $\delta^*(y, a, b, \omega)$, $a^*(y, a, b, \omega)$ and $b^*(y, a, b, \omega)$ be the policy functions for default, reserves and debt for the next period. The world risk-free interest rate r is exogenous. The zero-profits condition of lenders is:

$$q(y, a', b', \omega) = (1 + r_f)^{-1} \, \mathbb{E}_{y, \omega} \bigg[\big(1 - \delta^*(y', a', b', \omega') \big) \times \ldots$$

$$\Big[\lambda + (1 - \lambda)z + (1 - \lambda)q\big(y', a^*(y', a', b', \omega'), b^*(y', a', b', \omega')\big) \Big] \bigg]. \tag{6}$$

2.3 Equilibrium

A Markov Perfect Equilibrium in this model consists of value functions $W^+ : S \to \mathbb{R}$, $V^+ : S \to \mathbb{R}$, $V^- : S \to \mathbb{R}$, $X : S_b \to \mathbb{R}$, and functions $a^* : S \to \mathbb{R}$, $b^* : S \to \mathbb{R}$ and $\delta^* : S \to \mathbb{R}$ and a price function $q : S \to \mathbb{R}$ defined over the state space $S = \mathbb{R}^3_+ \times \{0, 1\}$ such that:

1. Given the unconditional value function W, the default value function X solves the subproblem (2).

2. Given the price schedule $q(\cdot)$, the unconditional value function W and the default value function X, the value function V^+ solves subproblem (1).

3. Given the price schedule $q(\cdot)$, the unconditional value function W and the default value function X, the value function V^- solves subproblem (3).

4. Given the policy functions a^*, b^*, δ^* the price function q satisfies (6).

3 The Model's Mechanism

3.1 Description of the mechanism

A key aspect of the model is the specification of the rollover crises, which splits the state space into three regions. An upper region where default is never optimal even if there is no new lending, a bottom region where default is always optimal no matter the market conditions, and a middle region where repayment is optimal if and only if there is new lending. In this middle region the sunspot variable acts as a coordinating device that determines if there is a self-fulfilling debt crises ($\omega = 1$) or not ($\omega = 0$). As a result, the probability of facing a self-fulfilling debt crisis is the product of two probabilities: the probability that the sunspot variable takes the value $\omega = 1$ and the probability that the economy's fundamentals land in the vulnerability region.

The effect of adding reserves to the standard Eaton-Gersovitz setup with one-period debt is as follows: take as given the country's net foreign asset position, which is equal to reserves minus debt (i.e. $\text{NFA} = a - b$). Since reserves cannot be confiscated, it is known that an increase in reserves increases the value of default $X(y, a, \omega)$ for sure. Assuming the coupon z is such that debt is issued at par, an NFA neutral increase in reserves is matched with an increase in debt of equal size, and causes a decrease in the value of repayment $V^+(y, b, a, \omega)$ next period. This happens because the sovereign loses the difference between the return on those assets. Hence, an NFA neutral increase in reserves shifts resources from repayment to default states, and increases the default probability next period. The resource shifting is undesirable ex-ante because resources are more scarce on repayment states. In addition, the higher default probability implies a lower revenue on issuance. Putting those two together it follows that an NFA neutral increase in reserves is undesirable, which implies no reserves in equilibrium, consistent

with the findings of Alfaro and Kanczuk (2009).

The key channel in this paper is that reserve holdings also increase the value of repaying when there is no new lending $V^-(y, b, a, \omega)$. This happens because extra reserves can be used to service the additional debt and, with long debt maturity, some will be left for consumption smoothing. This makes repayment the dominant strategy for the government in more states of the world, precluding the bad equilibrium of no-lending and default in those states. Hence, more international reserves shrink the vulnerability region of the state space (the region in which $V^- < X < V^+$). From an *ex-ante* perspective, more reserves reduce the probability of landing in that region, and thus the probability of a self-fulfilling crisis is reduced.

It is possible to shed further light on the mechanism by which reserves reduce the likelihood of self-fulfilling debt crises, and the conditions that allow the model to support a portfolio with debt and reserves, by assuming that the value functions, policy functions and bond pricing function are differentiable, which is done in this section. This is not necessary for the quantitative solution method to work.

3.2 Default thresholds

For a given debt, reserves and sunspot realization (a, b, ω), define the default thresholds $d^f(a, b, \omega)$ and $d^s(a, b, \omega)$, depicted on Figure 3, as the income realization that leaves the sovereign indifferent between repaying and defaulting. They are defined implicitly by:

$$V^+\big(d^f(a, b, \omega), a, b, \omega\big) = X\big(d^f(a, b, \omega), a, \omega\big), \tag{7}$$

$$V^-\big(d^s(a, b, \omega), a, b, \omega\big) = X\big(d^s(a, b, \omega), a, \omega\big). \tag{8}$$

For the case when there are no self-fulfilling debt crises ($\omega = 1$), figure 3a depicts the value after default $X(\cdot)$ and after repayment $V^+(\cdot)$ as a function of the income realization y. As it is standard in the sovereign default literature, the default output cost function is set to be increasing (and convex) in output to match the fact that default tends to happen for low realizations of the income, which implies the graph of $X(\cdot)$ is above the graph of $V^+(\cdot)$ for low realizations of output. The crossing point determines the *fundamentals* default threshold d^f. A fundamentals default occurs if $y < d^f(a, b, \omega)$. The unconditional value function $W(\cdot)$ is just the upper envelope of X and V, also depicted in Figure 3a.

On the other hand, when $\omega = 0$ the value after repayment but without new lending $V^-(\cdot)$ plays a role. In Figure 3b the graph of $V^-(\cdot)$ is depicted along those of $V^+(\cdot)$ and

$X(\cdot)$. For high income realizations the no-lending constraint does not bind, since the sovereign finds optimal to buy back debt, hence $V^+(\cdot) = V^-(\cdot)$. But as income decreases, the constraint starts binding and V^- decreases faster than V^+.

Figure 3: Default thresholds for a fixed debt-reserves pair.

(a) $\omega = 1$ (b) $\omega = 0$
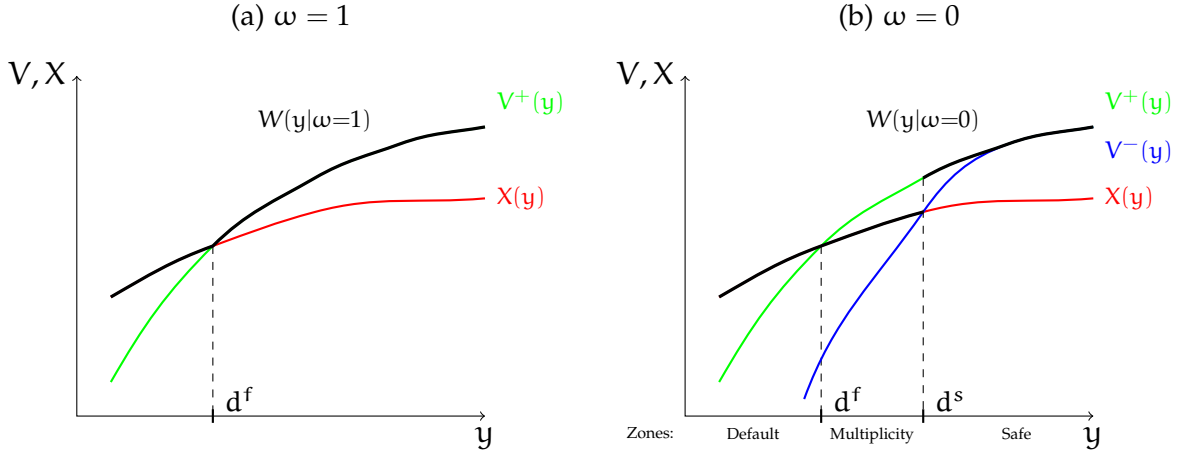
The crossing point between V^- and the default value X determines the *Self-fulfilling crises* default threshold d^s. A self-fulfilling default occurs if $\omega = 0$ and $d^f(a, b, \omega) < y < d^s(a, b, \omega)$. Each point $s = (y, a, b, \omega)$ of the state space is classified to be in one of three regions, also depicted in Figure 3b:

i) Safe region: when $y > d^s(a, b, \omega)$, repayment is the dominant strategy for the sovereign no matter what the lenders do and thus the state is part of the safe region.

ii) Default region: When $y < d^f(a, b, \omega)$, default is the best option for the government given the weak fundamentals and thus the state is part of the default region. Defaults occurring in this region are denominated fundamentals defaults.

iii) Multiplicity: When $d^f(a, b, \omega) < y < d^s(a, b, \omega)$, the state is in the multiplicity region. Here default happens if and only if $\omega = 0$, and when it happens it is denominated a self-fulfilling default.

Note that the unconditional value function $W(\cdot)$ when $\omega = 0$ is discontinuous at the self-fulfilling default threshold $y = d^f$. This is because of the outcome of the game of default and rollover described in Section 2.2. The value V^- is never realized on the equilibrium path, only V^+ or X are realizations of Nash equilibria.

Given the thresholds, the probability of default next period can be written as:

$$\Pr(\text{def}) = \Pr\big(\delta(y', a', b', \omega') = 1\big) = \Pr(y' < d^f) + \Pr(\omega' = 0)\Pr(d^f < y' < d^s). \quad (9)$$

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And the equilibrium bond price function can be compactly specified as:

$$q(1+r) = [1 - \Pr(\text{def})]\Big((\lambda + (1-\lambda)z) + (1-\lambda)\,\mathbb{E}_{y,\omega}[q'|\text{repay}]\Big). \qquad (10)$$

With the features of the model defined, the mechanism can be stated briefly: issuing extra debt to finance reserves accumulation, increases the fundamentals default threshold but reduces the self-fulfilling one. In aggregate, it reduces the next period default probability and thus the sovereign spread. These statements are developed in the next subsection.

3.3   Consolidating debt and reserves

Assume that in an equilibrium the economy is at a state $s = (y, a, b, \omega)$ in which both chosen reserves and debt are positive $a^*(s) > 0$, $b^*(s) > 0$ and there is repayment. After repayment is chosen, define a "consolidation of debt and reserves" as the following portfolio adjustment made at the current equilibrium debt price $\bar{q} = q(y, a^*(s), b^*(s), \omega)$: the sovereign issues ε less of face-valued debt and decreases reserves by $\bar{q}(1+r)\varepsilon$ tomorrow. To keep continuation values accounting easy, this consolidation is fully undone in the next period, that is: for each realization of the exogenous states $(y'\omega')$ in the next period, the asset holdings chosen for two periods ahead are the same before and after the consolidation.

Next, it will be argued that the consolidation operation is always beneficial to the sovereign in the standard Eaton-Gersowitz setup where no self-fulfilling crises occur, but that it can leave the government worse off once self-fulfilling crises are possible. To determine the overall effect of consolidation, the impact on current consumption and on next period consumption has to be addressed.

Consolidating debt and reserves is not neutral on current consumption since bond prices react. How do they react depends in part on how the thresholds in the next period move. This in turn depends on the consolidation effect on next period consumption since continuation values two periods ahead are kept constant.

In the next period, the consolidation effect on available resources is as follows. In the states in the default region, reserves change by

$$- \bar{q}(1+r)\varepsilon \qquad (11)$$

and this is the total effect since debt is wiped away.

The effect on resources under repayment has two components: the first one comes

from the fall in reserves (equation 11), the second one comes from the reduction in debt outstanding

$$\varepsilon\left[\lambda + (1-\lambda)(z + q')\right],$$

it includes the lower debt service $\lambda + (1-\lambda)z$ and the extra room for new issuance $(1-\lambda)q'$.

Using condition (10) for $\bar{q}$, the total effect on resources after repayment is then:

$$\varepsilon\bar{q}(1+r)\frac{\Pr(\mathtt{def})}{1-\Pr(\mathtt{def})} + \varepsilon(1-\lambda)\left(q' - \mathbb{E}_{y,\omega}[q'|\mathtt{repay}]\right). \tag{12}$$

The consolidation effect on resources next period is an actuarially fair transfer. The first term of equation (12) captures the average transfer into each of the repayment states, which is just the negative of the average transfer out of each of the default states (equation 11) times their relative masses.

The consolidation operation keeps fixed the debt outstanding two periods ahead (b''), but there is a change in the periods that debt is issued. After the consolidation, some debt is issued in the next period instead of the current one. The second term on equation (12) captures the fact that market conditions may change between these two periods and, depending on the price schedule, it may or may not be cheaper to issue in the current period and save as reserves than to issue in the next period. In the current period, lenders anticipate this potential for capital gains or losses and adjust the price accordingly such that on expectation it vanishes away. Hence the second term is just a zero-sum reshuffling of resources among repayment states.

The effect on resources is actuarially fair, but since the sovereign is risk averse, there is room for utility changes. The expected marginal change in utility next period from consolidation can be decomposed in four parts:

1. The expected utility loss in the default states because of less resources available: $-\Pr(\mathtt{def}) \cdot \left(\varepsilon\bar{q}(1+r)\,\mathbb{E}\left[u_c(c'_x)|\mathtt{def}\right]\right)$

2. The expected utility gain in repayment states coming from the net resource transfer from the default states: $\left(1 - \Pr(\mathtt{def})\right) \cdot \varepsilon\bar{q}(1+r)\frac{\Pr(\mathtt{def})}{1-\Pr(\mathtt{def})}\,\mathbb{E}\left[u_c(c')|\mathtt{repay}\right]$

3. The expected utility gain or loss coming from shifting the issuance period of the surviving debt: $+\varepsilon(1-\lambda)\mathrm{Cov}\left(u_c(c'), q'\right)$,

4. The expected utility gain or loss coming from the change in the self-fulfilling de-

fault threshold.

$$-\Pi_\omega(\omega'=0)\big[V^+(d^s,a',b',0)-X(d^s,a',b',0)\big]f(d^s|y)\Delta d^s, \qquad (13)$$

where c_x is the consumption policy in case of default and Δd^s is the change in the self-fulfilling default threshold due to the consolidation operation. This fourth term appears because of the discontinuity of the function $W(\cdot)$ when $\omega=0$, which happens because at the self-fulfilling default threshold the sovereign is *not* indifferent between repay and default.

Let $z = \varepsilon\bar{q}(1+r)\Pr(\mathrm{def})$. Abstracting from the effect coming from the change in thresholds, the marginal effect of consolidation on utility next period can be written as:

$$-z\,\mathbb{E}\big[u_c(c'_x)|\mathrm{def}\big] + z\,\mathbb{E}\big[u_c(c')|\mathrm{repay}\big] + (1-\lambda)\mathrm{Cov}\big(u_c(c'),q'\big). \qquad (14)$$

3.3.1 The case of one-period debt

When $(\lambda = 1)$ the resource transfer into repayment states (equation 12) reduces to $+\varepsilon\Pr(\mathrm{def})$. Those resources come from the default states, $-\varepsilon(1-\Pr(\mathrm{def}))$ (equation 11).

The fact that resources increase in repayment states and decrease in default directly implies lower default thresholds. Consequently, bond prices move in favor of the sovereign and, since $\Delta d^s < 0$, there are gains from enlarging the safe zone (equation 13). However, the benefit of this bond appreciation should be compared against the expected utility cost (or benefit) of that transfer of resources.

Given that reserves are a risk-free asset and debt provides some insurance against low realizations of income by the means of default, it is reasonable to think that in equilibrium the expected marginal utility under default is the same as the expected marginal utility under repayment: that is $\mathbb{E}\big[u_c(c'_x)|\mathrm{def}\big] = \mathbb{E}\big[u_c(c')|\mathrm{repay}\big]$. However this is not the case in the Eaton-Gersovitz framework because the lack of commitment: the government cannot choose separately the default and repayment states and the amount borrowed, because more borrowing implies more default states. In appendix A.3 the sovereign's problem with commitment is stated along the lines of Rios-Rull and Mateos-Planas (2016) and the first order condition showing $\mathbb{E}\big[u_c(c'_x)|\mathrm{def}\big] < \mathbb{E}\big[u_c(c')|\mathrm{repay}\big]$ is presented.

So far it has been established that in the case of one-period debt, the consolidation of debt and reserves decreases default thresholds in the next period, which implies a better price schedule in the current period and some utility gain in the next period

coming from those states getting out of the multiplicity zone. It also moves resources from the default states to the repayment states, which is *ex-ante* desirable because the lack of commitment forces the sovereign to default in more states than what it would have committed to. All that adds up to debt and reserves not coexisting in equilibrium, because the consolidation increases not only current but also expected utility. This is consistent with the results found by Alfaro and Kanczuk (2009).

3.3.2   The case of long-term debt without self-fulfilling crises

In this case, the only threshold that matters is the one for fundamentals default. The change in that threshold depends on the change of the value functions at the old threshold, more precisely the variation in $X(d^f)$ minus the change in $V^+(d^f)$, due to the consolidation operation:

$$\Delta X(d^f) \simeq -\varepsilon u_c(\check{c}'_x)\bar{q}(1+r), \tag{15}$$

$$\Delta V^+(d^f) \simeq \varepsilon u_c(\check{c}') \Big(\frac{\bar{q}(1+r) \cdot \Pr(\text{def})}{1 - \Pr(\text{def})} + (1-\lambda)\big(\check{q}' - \mathbb{E}_{y,\omega}[q'|\text{pay}]\big) \Big), \tag{16}$$

where a variable with a ˇ indicates it is evaluated at the threshold.

Figure 4 depicts the effect of consolidation on this threshold, on the same plane of income and utility used in Figure 3a. The consolidation effect on the value of default is the same as in the case with one-period debt: less reserves imply a reduction for all future states. It can be seen in Figure 4 as a new graph for the value function X' (in red) drawn below the original curve X (in gray).

As stated before, there is a net transfer of resources into repayment states (first term of equation 12). However, the valuation effect on resources $(1-\lambda)\big(q' - \mathbb{E}_{y,\omega}[q'|\text{repay}]\big)$ is negative for low income realizations like $y = d^f$, even though on average it is zero. This implies that ΔV^+ may be negative. Numerically, this effect is small and the change in V^+ at the threshold is still greater than ΔX. Hence, the threshold d^f goes down which makes the bond price in the current period to increase. Figure 4 depicts the new curve V'^+, which is close to the original graph of V^+.

Regarding the change in utility in the next period, the inequality $\mathbb{E}\big[u_c(c'_x)|\text{def}\big] < \mathbb{E}\big[u_c(c')|\text{repay}\big]$ still holds. The same argument from the one-period case holds here: the lack of commitment implies a trade-off between the default threshold next period and the resources brought into the current period. [12] However, in this case the covariance

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Figure 4: Consolidation effect on d^f threshold
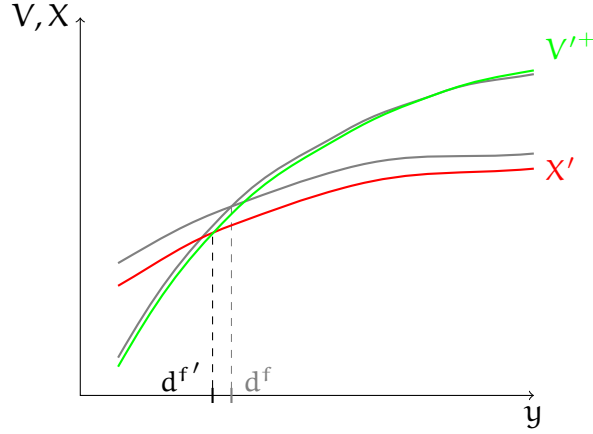
term of the consolidation effect in expected utility (equation 14) is negative. This could potentially generate an incentive for reserves holding, although there is an extra cost of long-term debt, which is that lenders now incorporate dilution risk in the price schedule. Numerically, the gains in utility due to the covariance are of second order and less relevant than the losses due to the lack of commitment, which are of first order.

Hence the main conclusion still is that the consolidation operation reduces the default threshold, which implies higher bond prices, and shifts resources into repayment states, which is desirable. It also reshuffles resources among repayment states, which may hurt the sovereign. Numerically the first two effects dominate and the consolidation operation increases the value for the government, which implies no debt and reserves coexistence.

3.3.3 The case for reserves with self-fulfilling crises

In this case, the behavior of the fundamentals default threshold is the same described for the case without self-fulfilling crises. The difference now is that the behavior of the self-fulfilling default threshold matters. The consolidation effect on d^s depends on the change in the value of repayment with *no* rollover V^-:

$$\Delta V^- = -\varepsilon \bar{q} R V_a^- - \varepsilon V_b^- = \varepsilon u_c(c') \Big[- \bar{q} R + \big(\lambda + (1-\lambda)(z + q') \big) \Big] - \mu(1-\lambda), \qquad (17)$$

where μ is the multiplier of the no-rollover constraint $b' \leqslant (1-\lambda)b$. As in the previous case, the term in square brackets is still generically positive. However the multiplier term is of first order as long as $\lambda < 1$. Directly, less debt outstanding is worse in case of a rollover crisis because it tightens the borrowing constraint. Indirectly, a lower amount

reserves implies less room to smooth consumption when additional borrowing is not possible, which increases the value of the multiplier. Hence V^- can fall significantly with the consolidation operation.
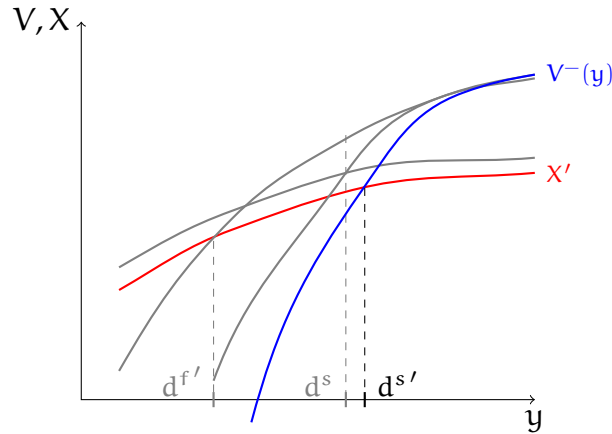
Figure 5: Consolidation effect on d^s threshold

Figure 5 graphs new V'^- relative to the old one depicted in Figure 3b. The bigger the gap between the old V^+ and V^- is before the consolidation operation, the more binding the constraint is and the bigger the fall in V'^- after the consolidation.

Evaluated at the d^s threshold, the fall in V^- due to the lower reserves holdings will generally be greater than the reduction in the default value X due to the same cause. In both cases, more borrowing is not possible, but in default there is no need for debt servicing and the output loss is small since it happens at low income realizations. Hence, available resources are more tight under repayment and no rollover, which implies the self-fulfilling threshold d^s increases. This increase in the threshold is depicted in Figure 5.

3.4 The role of reserves in preventing self-fulfilling debt crises

Given that consolidation increases the self-fulfilling default threshold, it has the potential to be very damaging, since that increase translates into discrete expected utility losses in some states in addition to bond price reductions due to the increased probability of default. In this subsection an operation opposite to consolidation is considered: a debt financed reserve holdings increase.

The discussion in the previous subsection still applies but the effects on the thresholds are the opposite. In particular, debt financed reserves increase the fundamentals default threshold d^f but they reduce the self-fulfilling threshold d^s. Figure 6 shows the

effect of this operation on both default thresholds, which is the inverse of reserves-debt consolidation. It abstracts from the value function curves and focuses only on the *x*-axis of Figures 4 and 5.

Figure 6: Change in zone thresholds from a debt financed reserves increase
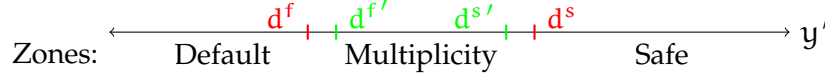
The debt financed accumulation of reserves shrinks the multiplicity zone and enlarges both the default and safe zones. The effect on default and bond prices is harder to isolate, but some intuition can be given. In the long run, default rates are low which implies the sovereign avoids falling into default. There is little mass in the default region, most of it is in the safe one. The probability of default next period is:

$$\Pr(\delta' = 1) = \Pr(y' < d^f) + \Pr(\omega' = 0)\Pr(d^f < y' < d^s) \tag{18}$$

Hence, when taking expectations *ex-ante* the effect of the enlarging safe region dominates. The *future* probability of default falls and that causes *current* spreads to go down as well.

Taking stock: the previous discussion outlined the mechanism through which international reserve holdings can reduce sovereign spreads and the probability of self-fulfilling debt crises. However, the model does not yield unambiguous predictions about the strength of the mechanism driving the link between reserves, self-fulfilling crises and spreads, and the overall direction of the effects connecting these variables. For that reason the numerical predictions of the model are studied in the next section.

4   Quantitative analysis

4.1   Solution Method and Calibration

The model is solved on a discretized state space using the method proposed by Chatterjee and Eyigungor (2012). [13] Gordon and Guerrón-Quintana (2013) develop a variant of

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[13]Which is the best algorithm for computing a numerical solution of sovereign default models with long term debt is an open question. As mentioned in Chatterjee and Eyigungor (2012), with long term debt the budget sets for the sovereign are not convex, hence infinitesimal changes in the value function can lead to large changes in policies, causing convergence problems in global methods. They introduced a small *purification* i.i.d. endowment shock to smooth the value and policy functions. Under monotonicity assumptions, they are able to guarantees the existence of an equilibrium on discretized state spaces.

that algorithm to deal with multiple assets in an efficient manner, reducing the dimensionality curse of endogenous states.[14]

Debt and reserves come from equally spaced grids, with 100 points each. Log-output is assumed to follow an AR(1) process:

$$\ln(y_t) = \rho \ln(y_{t-1}) + v_t,$$

where $v_t \sim \mathcal{N}(0, \sigma_v)$. The endowment process is discretized using Rouwenhorst's method to a grid of 50 points. Kopecky and Suen (2010) found this method to have better properties for highly correlated processes.

The utility function is the standard CRRA:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

The function that characterizes the income cost of default is the same as in Chatterjee and Eyigungor (2012)::

$$\phi(y) = \max\{0, d_0 y + d_1 y^2\}$$

where $d_0 \in \mathbb{R}$ and $d_1 \geqslant 0$. Note that d_0 can take negative values, which implies zero cost for low income realizations. [15]

The sunspot variable $\omega \in \{0, 1\}$ is assumed to follow a two-state Markov chain, with transition matrix:

$$\Gamma = \begin{bmatrix} \Gamma_{11} & 1 - \Gamma_{11} \\ 1 - \Gamma_{22} & \Gamma_{22} \end{bmatrix}.$$

Two key parameters in this model are the Γ_{11} and Γ_{22}. Very little is known about them because they are not directly observable and very hard to estimate from data. A stance is taken here to pin them down for a baseline calibration, but not without acknowledging that the values found should not be taken as precise estimates.

The model is calibrated to Mexican data, in line with the recent literature on international reserves that focuses on Mexico (Bianchi et al., 2012; Tavares, 2015). Although the sovereign default literature generally uses Argentina, the fact that it had a fixed exchange rate until 2001 implied its reserve holdings had to back the money in circulation,

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[14]For a given state, the method by Chatterjee and Eyigungor (2012) finds the output default threshold and also the thresholds for portfolio allocation by comparing all the feasible choices. Gordon and Guerrón-Quintana (2013) variant proposes a way to efficiently discard big portions of the portfolio state space.

[15]Arellano (2008) introduced a *kinked* specification for the default cost, and showed it is necessary to prop up the sustainable debt and to make default occur in bad times. Chatterjee and Eyigungor (2012) show that the quadratic form allows this models to better match the spread's standard deviation.

making the precautionary-savings motive for reserves of second order.

Unless stated otherwise, the data covers the period between 1994:Q1 and 2015:Q4. The model has 12 parameter values to select. Seven of these parameter values can be set directly using the data. The corresponding parameter values are shown in Table 3.

The bond duration parameter λ is set to $20^{-1} = 0.05$ yielding an average maturity of 20 quarters (5 years). Broner, Lorenzoni, and Schmukler (2013) find an average maturity of 10 years *at issuance* which implies an average *outstanding* debt maturity of 5 years. The interest rate spread on debt is taken from J.P. Morgan Emerging Market Bond Index (EMBI+). The average *annualized* blended spread is 228 basis points.

The quarterly real risk-free rate r is set to 0.38%, which is one quarter of the average nominal yield on 2 year constant maturity treasury bills deflated by the US PCE, using data from the St. Louis FRED database. Two reasons motivate this choice: first, no less than 70% of international reserves were invested at maturities longer than 1 year between 2007 and 2010 (McCauley and Rigaudy, 2011); second, the EMBI spread is calculated as the yield difference against a portfolio of US bonds of similar duration, which justifies the use of longer maturities.

The quarterly coupon z is set to 0.95%, which is the risk-free rate plus one quarter of the annualized average spread (228 bp). This parameter just scales up or down the cash-flow associated with debt but it is useful to make the debt stock in the model easier to compare with that in the data. Sovereign debt in the real world is measured at face value, and interest payments only enter the debt stock statistics when they are due, hence it is useful to have both -the market value and face value of debt- coincide (the latter is 1). This happens when the coupon is exactly the risk-free rate plus the spread. In addition, is common practice among sovereign issuers to set the promised coupons in their bond auctions aiming to sell debt at par.

The parameters ρ and σ_ν that govern the endowment are set to match the cyclical properties of Mexican real GDP times the real exchange rate. This captures not only real output volatility but also changes in the debt burden arising from real exchange rate fluctuations. [16] Hence, the Mexican GDP in US dollars is first deflated with the US PCE

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[16]The sovereign default literature has mainly focused on Argentina, which has a very volatile real income process, with some success (Arellano, 2008; Chatterjee and Eyigungor, 2012). Recently, Aguiar et al. (2016b) showed the struggles Eaton-Gersovitz models face when trying to match the quantitative success of those models on Mexico, which has a much less volatile real income process. One overlooked difference among the two economies is the exchange rate arrangement: Mexico has a floating exchange rate while Argentina had a fixed one until to its most recent default episode. Floating exchange rates are known to be a buffer to absorb external shocks, which can account for the lower volatility of Mexican real income relative to the Argentinian one. However, floating exchange rates expose the sovereign to the currency mismatch problem, as the exchange rate volatility translates into debt burden volatility relative to income. Eichengreen, Hausmann, and Panizza (2007) discuss how international reserve holdings can

index to leave it in global basket consumption terms. Then, an AR(1) process is fitted to the Hodrick-Prescott detrended series. The parameters for the income autocorrelation and the standard deviation of the income shock are $\rho = 0.76$ and $\sigma_v = 6.0\%$.

The parameter ε governing the re-entry probability is set at 0.128 which implies an average exclusion time of 8 quarters. This is consistent with the findings of Gelos, Sahay, and Sandleris (2011) for the average exclusion time for default episodes in the 90's. Also, as standard in the sovereign debt literature, the relative risk aversion coefficient is set to 2.

Table 3: Parameters related to moments in the data

Parameter	Value	Data moment	Source
r	0.038	Avg. 2yr US Treasury real yield	FRED
λ	0.05	Govt. external debt duration	Broner et al. (2013)
z	0.096	r plus EMBI spread	Datastream
ρ	0.76	GDP autocorrelation	WB-GEM
σ_v	0.06	GDP standard dev.	WB-GEM
γ	2	Standard in literature.	-
ε	0.125	Avg. default exclusion time	Gelos et al. (2011)
Γ_{22}	0.875	Avg. duration of SS events	Jeanne and Rancière (2011)

The transition probability parameters governing the sunspot Markov process determine the frequency and duration of self-fulfilling debt crises. The former is hard to measure directly in the data since those crises are only observed when the sovereign is in the multiplicity zone. On the other hand, the duration of systemic sudden stops can be observed in the data. Bianchi et al. (2012) report a sudden-stop duration of 1.12 years which implies $1 - \Gamma_{22} = \frac{1}{4 \cdot 1.12} = 0.22$ and Jeanne and Rancière (2011) find a duration of 4 years which would imply $1 - \Gamma_{22} = \frac{1}{4 \cdot 4} = 0.032$. The baseline calibration is set to the middle point of those estimates $1 - \Gamma_{22} = 0.125$ which implies a duration of 2 years.

The remaining four parameter values: β, d_0, d_1 and Γ_{11} are set using a Simulated Method of Moments (SMM) algorithm.

In the sovereign debt literature, the goal is to match the mean debt to GDP ratio, the average spread and the standard deviation of the spread. As argued by Aguiar et al. (2016b), the first three parameters have been found to be enough to match the debt level and average spread, but they find that either the spread volatility is too low or the correlation between the spread and income predicted by the model is very high. In this framework, the possibility of self-fulfilling crises allows multiple bond price schedules to

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lessen the currency mismatch problem.

Table 4: Parameters set by SMM

Parameter	Value	Targeted Moment
β	0.983	Debt to GDP ratio
d_0	-0.460	Default frequency
d_1	0.595	Spread Std. dev.
Γ_{11}	0.949	Spread-GDP corr.

be consistent with the same fundamentals, weakening the correlation between those fundamentals and sovereign spreads. Building of this, the calibration targets the correlation between spread and output cycle.

In addition, Aguiar et al. (2016b) document the role of risk premium as a driver of sovereign debt returns. They point to a gap between the realized returns on the EMBI+ index and the return on US govt indexes of similar maturity. Given that in this framework the lenders are assumed to be risk neutral, spreads are going to be closely related with the default probability. In the SMM calibration, instead of the spread, the target is the default probability which Aguiar et al. (2016b) find at 2% per year for Mexico.

Table 4 presents the values found during the procedure. The discount factor $\beta = 0.983$ corresponds to a yearly discount factor of 0.934 which is in line with the values found in the literature. The default cost parameters $d_0 = -0.46$ and $d_1 = 0.59$ imply a 15.6% proportional default cost when output is at its long run average, 8.1% when it is one standard deviation below average and 0.7% when it is two standard deviations below average.

The parameter governing the frequency of the sunspot switching from the good equilibrium to the bad one (a *panic*) obtained was $1 - \Gamma_{11} = 0.051$. It implies that those events happen roughly $\frac{1}{0.051} \simeq 20$ quarters (5 years). This parameter together with the probability of going back to the good equilibrium $1 - \Gamma_{22} = 0.125$ implies that financial markets are in the high beliefs regime $\frac{0.051}{0.051 + 0.125} = \simeq 28.5\%$ of the time.

Aguiar, Chatterjee, Cole, and Stangebye (2016c) estimated a two-state regime-switching model of Mexico's EMBI+ and found a transition of 0.12 from the low bond price into the high bond price regime, which is similar to the corresponding value used in the baseline calibration $1 - \Gamma_{22} = 0.125$. They also report a transition probability of 0.028 from the high bond price regime into the low bond price one, which implies a bad regime frequency of once every $\frac{1}{0.028} \simeq 36$ quarters (9 years), as they identified two episodes in their 20 year sample when the spread jumped more than what was granted by fundamentals. In the calibration here, the probability of the sunspot switching to the self-fulfilling cri-

ses *prone* regime is higher (0.051), but it is important to stress that in this framework not all periods where the sunspot variable point to the bad regime imply a spike in spreads since the economy's fundamentals can be deep inside the safe zone.

4.2 Results

Table 5: Data and model targeted moments

Targeted Moments	Data	Model
Debt to GDP ratio	15.8	15.9
Default probability	2.0	1.9
Std. dev. of spread	1.1	1.2
Spread-GDP corr.	-0.67	-0.68
Non-targeted Moments		
Reserves to GDP ratio	8.0	4.0
Average Spread	2.3	1.8

All in percentage points except Spread-GDP correlation which is scalar.

Table 5 presents the model's performance with respect to data moments. The model is able to match all the targeted moments, particularly the countercyclical and volatile spread together with observed debt holdings.

The model can explain half of the average reserves holdings: 4% of GDP in the model vs 8.0% in the data. This result leaves room for other motives for reserve accumulation. [17] On the spread side, the model is able to generate high mean spreads (177 b.p), but still they are 50 b.p. below the data average. As mentioned before, the lack of a risk premium on sovereign debt can explain this shortfall.

Figure 7 illustrates the effect of debt and reserve choices on sovereign spreads. The left panel depicts the spread schedule for different levels of future debt which has the standard convex shape reflecting higher spreads for higher debt positions. The right panel of figure 7 presents the spread schedule for reserve holdings choices. Consistent with the empirical evidence, reserves holdings are correlated with lower spreads.

Figure 8 presents the optimal debt and reserves policies as a function of current output. The state of the economy is assumed to be at the mean debt and reserve holdings and the sunspot variable pointing to no self-fulfilling crises ($\omega = 1$). The results show

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[17]It its important to acknowledge that there seems to be a recent upward trend in reserves accumulation in the data and a downward trend in the spread. A parameter shift can accommodate such trends, further research could delve into the precise nature of that shift and its quantitative preformance.

Figure 7: Effect of debt and reserves on spreads

(a) Next period Debt

(b) Next period Reserves
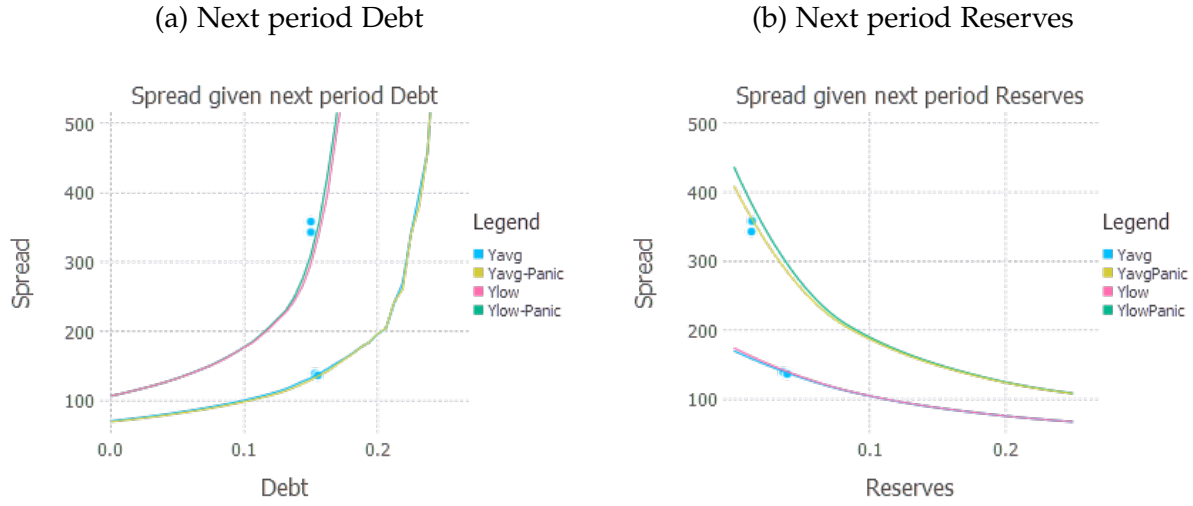
The current debt and reserve holdings are at their mean values. Yavg: mean income. Ylow: mean minus 1.6 standard deviations of income. Panic: $\omega = 0$.

Figure 8: Policy response to changes in current output

(a) Next period Debt

(b) Next period Reserves
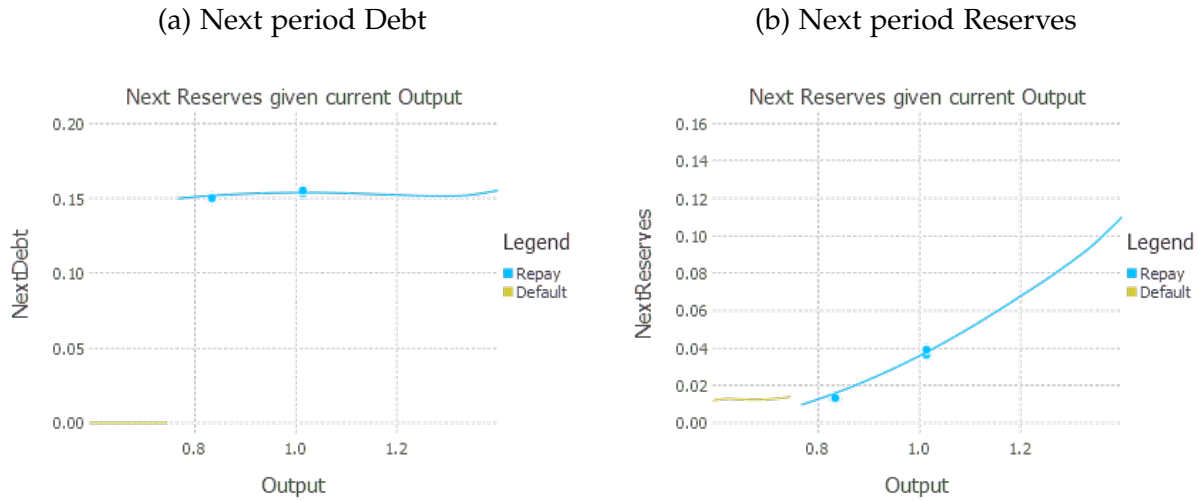
The current debt and reserve holdings are at their mean values. The sunspot variable value is $\omega = 1$.

that reserves are accumulated in periods of high output while the debt response is somewhat muted, which is consistent with the findings of Aguiar, Amador, Hopenhayn, and Werning (2016a). Aguiar et al. (2016a) argue debt buybacks are very costly because the bond prices move against the government and they find that adjustments are better performed in the short-term margin, which in this framework corresponds to the reserves margin. Debt and reserve accumulation are thus procyclical, which is consistent with the findings by Broner et al. (2013) on gross capital flows.

Table 6 presents the results of a regression of the sovereign spread on debt, reserves, current account, growth and the sunspot variable using data simulated from the model.

Table 6: Spread regression on model simulated data

Dep. var.: Spread	Pooled OLS β/s.e.
L.Reserves/GDP	-9.598***
	(0.066)
L.Debt/GDP	8.797***
	(0.109)
L. Sunspot	1.560**
	(0.749)
L.Current Account/GDP	-10.033***
	(0.188)
L.GDP growth	-4.005***
	(0.049)
Constant	83.872***
	(2.063)
R^2	0.325
N	96427

All variables are simulated from the model using the baseline calibration. "L." indicates one period lagged variable. Standard errors in parentheses. * Significance at the 10 percent level. ** Significance at the 5 percent level. *** Significance at the 1 percent level.

The results are very similar to those found in the data (Table 2). The reserves coefficient is negative and slightly larger in magnitude than the coefficient on debt, indicating that, in the model, higher levels of both debt and reserves are not associated with higher spreads.

The coefficient on the sunspot variable is small, which seems odd but in fact is consistent with the features of the model. First, in the safe and in the default region the sunspot is not very relevant, besides some impact through the expected value of landing in the multiplicity region in subsequents periods, which is small. Second, in the multiplicity region the direct effect of $\omega = 0$ is to cause a self-fulfilling crises where the country defaults, but those periods are dropped in the regression since the spread goes to infinity. Hence the coefficient in the regression shown in Table 2 is capturing just the indirect effect of the sunspot on spreads.

To further assess the impact of the sunspot variable and the proposed mechanism, Table 7 presents the results of alternative specifications in which either the frequency or the duration of the bad equilibrium regime is reduced.

Table 7: Alternative specifications of the sunspot process

Targeted Moments[1]	Data	Baseline[2]	Low frequency[3]	Low duration[4]
Debt to GDP ratio	15.8	15.9	15.7	15.5
Default probability	2.0	1.9	1.6	2.0
Std. dev. of spread	1.1	1.2	1.1	1.4
Spread-GDP corr.	-0.61	-0.60	-0.60	-0.59
Non-targeted Moments				
Reserves to GDP ratio	8.0	4.0	3.0	2.6
Average Spread	2.3	1.8	1.8	2.0

[1] All in percentage points except Spread-GDP correlation which is scalar.
[2] Baseline specification described in Tables 3 and 4. On average, the self-fulfilling regime happens once every 5 years and lasts 2 years .
[3] Low frequency specification: the self-fulfilling regime happens on average once every 8 year. Expected duration still 2 years.
[4] Low duration specification: the self-fulfilling regime is expected to last 1 year. On average occurs once every 5 years.

The Low Frequency specification in Table 7 shows the numerical performance of an specification in which the parameter Γ_{11} is set such that the sunspot variable switches to the self-fulfilling crises prone equilibrium once every 8 years (instead of the 5 years in the baseline calibration). This specification can still match the average debt level, the spread volatility and spread correlation with income, but it is not able to match the default probability which decreases from 2.0% to 1.6%. On the non-targeted moments, the spread is still 1.8% but reserves holdings fall to 3.0% (compared to the baseline at 4.0%). Making the coordination problems less frequent reduces the perceived risk of the multiplicity zone which in turn lowers the incentive to accumulate reserves, and also reduces the frequency of self-fulfilling defaults.

The last column in Table 7 shows the quantitative results of the Low Duration specification, in which the parameter Γ_{22} is set such that the sunspot stays in the self-fulfilling crises prone regime for 1 year on average (compared to 2 years on the baseline calibration). This specification matches the average debt level, the default probability and the correlation between spread and income, but yields a higher spread volatility (1.4%). The spread level attained is higher that the one in the baseline (2.0% vs 1.8%) but the reserve holdings fall significantly to 2.6% (baseline: 4.0%). Reducing the duration of the self-fulfilling regime means that in the case of repayment without rollover the govern-

ment expects to quickly regain access to borrowing, which implies that a lower amount of reserves is required to smooth consumption and service debt if that is the case. In addition, the lower duration of this regime reduces its expected impact, meaning that the value functions V^+ and V^- are closer, and the multiplicity region is then smaller.

To sum up, reducing either the frequency or the duration of the self-fulfilling crises prone regime significantly reduces the average reserve holdings the sovereign finds optimal. This shows the relevance of the mechanism in generating a role for reserves.

5 Concluding Remarks

This paper developed a quantitative model of sovereign default and international reserves to address the optimal portfolio choice of the government. The model embedded Cole-Kehoe style self-fulfilling crises in a standard Eaton-Gersovitz setup with long-term debt. In this framework lenders were risk-neutral, but the government faced a rollover risk arising from coordination failures on the lenders' side.

The mechanism described here explained the role of reserves in reducing spreads and the probability of debt crises. It was shown that for a given *Net Foreign Asset* position, additional reserves reduce the set of states that allow multiple equilibria regarding the repayment and rollover decisions. Higher reserve holdings preclude some of the risk of a self-fulfilling debt crises *in the future* and consequently reduce *current* sovereign spreads.

The model was calibrated to replicate the average external public debt, the default frequency and the volatility and countercyclicality of sovereign spreads. In the simulations, the model generates mean reserve holdings that are half as big as the observed one. In addition, the mean spread level in the model is 78% of the one observed in an environment with risk-neutral lenders.

This paper caught up with the empirical literature on vulnerability measures to sovereign debt crisis that has established the connection between higher reserve holdings and lower crises probability.

Further research can use the developed framework to evaluate policies aiming at preventing crises in sovereign debt markets. Those include the standard policy prescription rules, like the reserves adequacy ratios to imports, debt, output, short-term debt or debt service, or the more elaborated and widely used Guidotti-Greenspan rule.

In addition, this model is in position to evaluate the different contingent lending arrangements put in place by the IMF and other developed countries's central banks. Those are motivated as means to generate confidence in sovereign debt markets and

prevent self-fulfilling crises, and are not supposed to bail-out or subsidize lending to the recipient countries.

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Appendix A   Continuous State Space Heuristics

In this section it is assumed that the state space is continuous and that the value functions X, V^+ and V^- and their corresponding policy functions are continuously differentiable. The default decision follows from the thresholds defined in equation (7) and (8) and those are also assumed to be differentiable.

Given the default thresholds, the problem for the government after repayment can be written as:

$$
\begin{aligned}
V^+(y, a, b, \omega) = \max_{a', b', c} u(c) + \beta \Pi_\omega(\omega' = 1) &\left[\int_{-\infty}^{d_f(a',b',1)} X(y', a', 1) dF(y'|y) \right. \\
&\left. + \int_{d^f(a',b',1)}^{\infty} V^+(y', a', b', 1) dF(y'|y) \right] \\
+ \beta \Pi_\omega(\omega' = 0) &\left[\int_{-\infty}^{d_s(a',b',0)} X(y', a', 1) dF(y'|y) \right. \\
&\left. + \int_{d^s(a',b',0)}^{\infty} V^+(y', a', b', 0) dF(y'|y) \right]
\end{aligned}
\tag{19}
$$

$$
\text{s.t.} \quad c = y + a - \frac{a'}{1+r} + q(y, a', b', \omega)\left(b' - (1-\lambda)b\right) - \left(\lambda + (1-\lambda)z\right)b.
$$

The equilibrium bond pricing equation (6) is then:

$$
\begin{aligned}
q(y, a', b', w) = \frac{\Pi_\omega(\omega' = 1)}{1+r} &\left[\left(1 - F(d^f)\right)\left(\lambda + (1-\lambda)z\right) \right. \\
&\left. + (1-\lambda) \int_{d^f}^{\infty} q\left(y', a^*(s'), b^*(s'), 1\right) dF(y') \right] \\
+ \frac{\Pi_\omega(\omega' = 0)}{1+r} &\left[\left(1 - F(d^s)\right)\left(\lambda + (1-\lambda)z\right) \right. \\
&\left. + (1-\lambda) \int_{d^s}^{\infty} q\left(y', a^*(s'), b^*(s'), 0\right) dF(y') \right]
\end{aligned}
\tag{20}
$$

A.1 The change in thresholds with respect to debt and reserves

Using the implicit function theorem the impact of changes in reserves and debt on those thresholds can be written as:

$$\frac{\partial d^f(a,b,\omega)}{\partial i} = d_i^f(a,b,\omega) = -\frac{V_i^+ - X_i}{V_y^+ - X_y}\bigg|_{y=d(a,b,\omega)}, \tag{21}$$

$$\frac{\partial d^s(a,b,\omega)}{\partial i} = d_i^s(a,b,\omega) = -\frac{V_i^- - X_i}{V_y^- - X_y}, \tag{22}$$

where $i \in \{a,b\}$ and V_i^+ is the partial derivative of the value function with respect to i and the functions are evaluated at $y = d(a,b,\omega)$. Notice that the existence of the default threshold implies that at $y = d(a,b,\omega)$, $V_y^+ > X_y$

The envelope conditions yield:

$$X_a(y,a,\omega) = \frac{\partial X^((y,a,\omega)}{\partial a} = u_c(c_d) \tag{23}$$

$$V_a^+(y,a,b,\omega) = \frac{\partial V^+(y,a,b,\omega)}{\partial a} = u_c(c) \tag{24}$$

$$V_b^+(y,a,b,\omega) = \frac{\partial V^+(y,a,b,\omega)}{\partial b} = -u_c(c)\big(\lambda + (1-\lambda)(z+q)\big) \tag{25}$$

Next, the effect over the current default threshold of an infinitesimal change in both current debt and reserves is computed. If the ratio of the change in reserves to debt is $\alpha_1 : \alpha_2$ then the change in the fundamentals default threshold $\Delta(d^f) = \alpha_1 d_a^f + \alpha_2 d_b^f$ is:

$$\Delta(d^f) = (V_y^+ - X_y)^{-1}\big(\alpha_1 u_c(\check{c}_x) + u_c(\check{c})\big(-\alpha_1 + \alpha_2\lambda + \alpha_2(1-\lambda)(z+\check{q})\big)\big), \tag{26}$$

where $\check{z}$ denotes the value or policy function z evaluated at $s = (d^f(a,b,\omega),a,b,\omega)$ and c_x is the consumption policy in case of default.

The Self-Fulfilling default threshold change is

$$\Delta d^s = d_a^s + d_b^s = \frac{-V_b^- - V_a^- + X_a}{V_y^- - X_y} \tag{27}$$

The main difference is that the envelope condition for V_b^- now includes a multiplier term.

$$V_b^- = -u(c)[\lambda + (1-\lambda)z + (1-\lambda)\check{Q}'] + \mu(1-\lambda)$$

Cole and Kehoe (2000) main insight: long term debt prevents self-fulfilling crises. Higher

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λ relaxes the constraint.

A.1.1 NFA-neutral change in holdings

Increase both current debt and reserves by 1\$. The fundamentals default threshold change is

$$\Delta d^f = d_a^f + d_b^f = \frac{-V_b^+ - V_a^+ + X_a}{V_y^+ - X_y}.$$

Using the *envelope* conditions it reduces to:

$$\Delta d^f \left[u_c(\check{c}) - u_c(\check{c}_\delta) \right] = -u_c(\check{c})(1-\lambda) \left[1 - z - Q(d, a', b', \omega) \right] + u_c(\check{c}_\delta)$$

With one period debt ($\lambda = 1$), threshold moves up for sure. In the long term debt case ($\lambda < 1$), the term $\left[1 - z - Q(d, a', b', \omega) \right]$ is small since debt is issued on average at par.

A.2 The Generalized Euler Equation (GEE)

This subsection follows Rios-Rull and Mateos-Planas (2016) and derives the first order conditions for debt and reserves under repayment.

Using the envelope conditions. The first order conditions with respect to next period reserves (a') is

$$
\begin{aligned}
\frac{\partial V}{\partial a'} = & -u_c(c) \left[(1+r)^{-1} - q_{a'} \left(b' - (1-\lambda)b \right) \right] \\
& + \beta \Pi_\omega(\omega' = 1) \left[\int_{-\infty}^{d_f(a',b',1)} u_c(c_x') dF(y'|y) + \int_{d^f(a',b',1)}^{\infty} u_c(c') dF(y'|y) \right] \\
& + \beta \Pi_\omega(\omega' = 0) \left[\int_{-\infty}^{d_s(a',b',0)} u_c(c_x') dF(y'|y) + \int_{d^s(a',b',0)}^{\infty} u_c(c') dF(y'|y) \right. \\
& \left. + \left[V^+(d^s, a', b', 0) - X(d^s, a', b', 0) \right] f(d^s|y) d_{a'}^s \right]
\end{aligned}
\tag{28}
$$

The FOC with respect to next period debt b' is:

$$
\begin{aligned}
\frac{\partial V}{\partial b'} = &+ u_c(c)\Big[q(y,a',b',w) + q_{b'}\big(b' - (1-\lambda)b\big)\Big] \\
&- \beta \Pi_w(w'=1) \int_{d^f(a',b',1)}^{\infty} u_c(c')\big[\lambda + (1-\lambda)(z+q')\big]dF(y'|y) \\
&- \beta \Pi_w(w'=0)\Bigg[\int_{d^s(a',b',0)}^{\infty} u_c(c')\big[\lambda + (1-\lambda)(z+q')\big]dF(y'|y) \\
&\quad + \Big[V^+(d^s,a',b',0) - X(d^s,a',b',0)\Big]f(d^s|y)d_{b'}^s\Bigg]
\end{aligned}
\tag{29}
$$

A.2.1 The debt-reserves consolidation operation

The marginal effect of the consolidation operation at the price $\bar{q} = q(y,a',b',w)$ is $-\bar{q}R\frac{\partial V}{\partial a'} - \frac{\partial V}{\partial b'}$, where $R = (1+r)$ and this is:

$$
\begin{aligned}
&u_c(c)\Big[-\big(\bar{q}Rq_{a'} + q_{b'}\big)\big(b' - (1-\lambda)b\big)\Big] \\
&+ \beta\Pi_w(w'=1)\Bigg[-\bar{q}R\int_{-\infty}^{d_f(a',b',1)} u_c(c'_x)dF(y'|y) \\
&\quad + \int_{d^f(a',b',1)}^{\infty} u_c(c')\big[\lambda + (1-\lambda)(z+q') - \bar{q}R\big]dF(y'|y)\Bigg] \\
&+ \beta\Pi_w(w'=0)\Bigg[-\bar{q}R\int_{-\infty}^{d_s(a',b',0)} u_c(c'_x)dF(y'|y) \\
&\quad + \int_{d^s(a',b',0)}^{\infty} u_c(c')\big[\lambda + (1-\lambda)(z+q') - \bar{q}R\big]dF(y'|y) \\
&\quad + \Big[V^+(d^s,a',b',0) - X(d^s,a',b',0)\Big]f(d^s|y)(-\bar{q}Rd_{a'}^s - d_{b'}^s)\Bigg]
\end{aligned}
\tag{30}
$$

The first term captures the effect on current consumption due to the change in the bond price. The last term captures the discrete gains from states coming out of the multiplicity region. The other terms capture the effect on future consumption depending on the repayment or default decision. From the equilibrium price equation (6):

$$
\lambda + (1-\lambda)(z+q') - \bar{q}R = \frac{\Pr(def)}{1 - \Pr(def)}\bar{q}R + (1-\lambda)(q' - \mathbb{E}[q'|repay]),
$$

where $\text{Pr}(\text{def})$ is the probability of default next period. The consolidation effect on utility can be compactly written as:

$$u_c(c)\Big[-\big(\bar{q}Rq_{a'}+q_{b'}\big)\big(b'-(1-\lambda)b\big)\Big]$$
$$-\beta\text{Pr}(\text{def})\bar{q}R\,\mathbb{E}\big[u_c(c'_x)|\text{def}\big]+\beta\big(1-\text{Pr}(\text{def})\big)\,\mathbb{E}\left[\frac{\text{Pr}(\text{def})}{1-\text{Pr}(\text{def})}\bar{q}Ru_c(c')\Big|\text{repay}\right] \quad (31)$$
$$+\big(1-\text{Pr}(\text{def})\big)(1-\lambda)\,\mathbb{E}\Big[u_c(c')\big(q'-\mathbb{E}[q']\big)\Big]$$
$$+\beta\Pi_\omega(\omega'=0)\Big[V^+(d^s,a',b',0)-X(d^s,a',b',0)\Big]f(d^s|y)\big(-\bar{q}Rd^s_{a'}-d^s_{b'}\big)$$

Define the change in price $\Delta q = -\bar{q}Rq_{a'}-q_{b'}$ and the change in the self fulfilling threshold $\Delta d^s = -\bar{q}Rd^s_{a'}-d^s_{b'}$, then the consolidation marginal effect is:

$$u_c(c)\Delta q\big(b'-(1-\lambda)b\big)$$
$$-\beta\Pi_\omega(\omega'=0)\big[V^+(d^s,a',b',0)-X(d^s,a',b',0)\big]f(d^s|y)\Delta d^s$$
$$+\beta\text{Pr}(\text{def})\bar{q}R\Big(-\mathbb{E}\big[u_c(c'_x)|\text{def}\big]+\mathbb{E}\big[u_c(c')|\text{repay}\big]\Big) \quad (32)$$
$$+\beta\big(1-\text{Pr}(\text{def})\big)(1-\lambda)\text{Cov}\big[u_c(c')q'|\text{repay}\big]$$

This operation is not revenue neutral in the first period because the bond price reacts to it, and the first term in equation (32) captures this effect on current utility. The second term captures the direct effect of the change in the self-fulfilling default threshold d^s in expected utility, the fundamentals default threshold change has no first order effect since the unconditional value function W is continuous at that point, but when $\omega = 0$ there is a jump in $W(\cdot,0)$ at $y = d^s(a,b,0)$ which captures the fact that crossing the self-fulfilling default threshold causes a discrete loss for the sovereign.

The third and fourth term in equation (32) reflect the resource transfer from default to repayment states. If the sovereign had the opportunity to credible commit to default and repay in selected states while keeping the expected repayment constant, the term $\Big(-\mathbb{E}\big[u_c(c'_x)|\text{def}\big]+\mathbb{E}\big[u_c(c')|\text{repay}\big]\Big)$ would be zero. However, in the Eaton-Gersovitz environment, the expected payment choice and the default threshold choice are made together, at the commitment optimal default threshold, the expected payment is low, which means current revenues are low. The sovereign faces a trade-off between issuing more debt and bringing more resources to the current period and issuing less debt but keep the default threshold low and close to the commitment optimal level.

The last term captures the utility gains from shifting the issuance date of debt outstanding two periods ahead. Before the consolidation operation some of that debt was

issued on the current period, after the consolidation it is issued in the next period. As lenders are risk neutral, the current price is consistent with the average prices of that debt in the next period, but for the sovereign this matters because of risk aversion. Since default is more likely at low realizations of output, consumption and bond prices will be positively correlated, hence this covariance term is generally negative.

A.3   The sovereign's problem with commitment

Here, the sovereign problem with commitment is briefly discussed. This section follows closely Rios-Rull and Mateos-Planas (2016).

The setup is the following: at the beginning of the period, before the income shock is realized, the government enters with reserves a a promise to repay k in expected value and chooses a default threshold d^c, contingent consumption $c(y)$ and contingent portfolio positions $k'(y)$, $a'(y)$.

The previous realization of output y_{-1} matters only for the expectations. Given that, the problem for the government can be written in recursive form as:

$$V^c(y_{-1}, a, k) = \max_{a'(y), k'(y), c(y), d^c} \int_{-\infty}^{\infty} u\big(c(y)\big) dF(y|y_{-1})$$

$$+ \beta \int_{-\infty}^{d^c} X^c\big(y, a'(y)\big) dF(y|y_{-1}) + \beta \int_{d^c}^{\infty} V^c\big(y, a'(y), k'(y)\big) dF(y|y_{-1}) \qquad (33)$$

$$\text{s.t.} \quad c(y) = y + a - \frac{a'}{1+r} + \mathbb{1}(y > d^c)\left[\frac{k'(y)}{1+r} - \frac{k}{1-F(d^c)}\right].$$

where V^c is the value function contingent on current reserve assets a and a promised expected repayment k. X^c is the value after default and $\mathbb{1}$ is the indicator function.

The first order condition with respect to the default threshold d^c is:

$$0 = \frac{\partial V^c(y_{-1}, a, k)}{\partial d^c} = \beta X\big(d^c, a'(d^c)\big) - \beta V\big(d^c, a'(d^c), k'(d^c)\big)$$

$$- \frac{kf(d^c)}{\big(1 - F(d^c)\big)^2} \int_{(d')^c}^{\infty} u_c\big(c(y')\big) dF(y'|y), \qquad (34)$$

from this first order condition it follows that the commitment default threshold d^c is determined by:

$$\beta X^c\big(d^c, a'(d^c)\big) = \beta V^c\big(d^c, a'(d^c), k'(d^c)\big) + \frac{kf(d^c)}{\big(1 - F(d^c)\big)^2} \int_{(d')^c}^{\infty} u_c\big(c(y')\big) dF(y'|y). \quad (35)$$

The difference between this equation and the fundamental's default equation (7) is the third term, which is positive, indicating the sovereign commits to repay in states where *ex-post* it would prefer to default.

Next, the first order conditions with respect to reserves and promised payments are derived. First, the envelope conditions yield:

$$X_a^c(y_{-1}, a) = \frac{\partial X^c(y_{-1}, a, k)}{\partial a} = \int_{-\infty}^{\infty} u_c(c_x(y)) dF(y|y_{-1}) \tag{36}$$

$$V_a^c(y_{-1}, a, k) = \frac{\partial V^c(y_{-1}, a, k)}{\partial a} = \int_{-\infty}^{\infty} u_c(c(y)) dF(y|y_{-1}) \tag{37}$$

$$V_k^c(y_{-1}, a, k) = \frac{\partial V^c(y_{-1}, a, k)}{\partial k} = \frac{-1}{1 - F(d^c)} \int_{d^c}^{\infty} u_c(c(y)) dF(y|y_{-1}) \tag{38}$$

The first order condition with respect to the promised payment $k'(y)$ is then:

$$0 = \frac{\partial V^c(y_{-1}, a, k)}{\partial k'(y)} = \frac{u_c(c(y))}{1 + r} - \frac{\beta}{1 - F((d')^c)} \int_{(d')^c}^{\infty} u_c(c(y')) dF(y'|y). \tag{39}$$

This equation can be compared with the first order condition w.r.t. b' in the Eaton-Gersovitz setup (equation 29. First, it is important to note that there are no self-fulfilling crises with commitment. Second, since the borrowing decision is independent of the default decision, there is no price feedback on the amount borrowed (i.e. the term with q_b in equation 29). With commitment the expected value of debt service is higher.

The first order condition with respect to reserves $a'(y)$ is:

$$0 = \frac{\partial V^c(y_{-1}, a, k)}{\partial a'(y)} = -\frac{u_c(c(y))}{1 + r} + \beta \int_{-\infty}^{(d')^c} u_c(c_x(y')) dF(y'|y) + \beta \int_{(d')^c}^{\infty} u_c(c(y')) dF(y'|y) \tag{40}$$

Adding both first order conditions, it follows that:

$$0 = \beta \int_{-\infty}^{(d')^c} u_c(c_x(y')) dF(y'|y) - \frac{\beta F((d')^c)}{1 - F((d')^c)} \int_{(d')^c}^{\infty} u_c(c(y')) dF(y'|y), \tag{41}$$

and this can be rewritten as:

$$0 = \frac{\beta}{F((d')^c)} \left(\mathbb{E}\left[u_c(c_x)|def\right] - \mathbb{E}\left[u_c(c)|repay\right] \right) \tag{42}$$

Hence, with commitment, expected marginal utilities under default and repayment are equalized. In the standard Eaton-Gersovitz framework, the expected marginal utility

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under default is lower that the expected marginal utility under repayment because the sovereign is defaulting on states with higher income. This implies that on the margin, shifting resources from default states to repayment states is efficient and thus the term in the third line of equation (32), which is part of the consolidation effect on utility, is positive.

Appendix B   Computation method

As mentioned in Chatterjee and Eyigungor (2012), with long term debt the budget sets for the sovereign are not convex. This is consequence of the debt dilution problem: with one-period debt, if the government deviates from equilibrium into slightly lower debt levels, bond prices react immediately to indicate a lower default probability. On the other hand, with long-term debt, bond prices almost do not react to the low debt levels since it is expected that the normal level will be chosen in the next period and current bondholders will be diluted.

This same argument explains the convergence problems of global methods solving this problem, the bond price function may not react for infinitesimal changes of debt holdings until a discrete change at a default state happens. In addition, as seen in figure 3, the possibility of self-fulfilling crises make the unconditional value function W discontinuous which exacerbates the convergence problem of global iterative methods.

Chatterjee and Eyigungor (2012) introduced a *purification* i.i.d. endowment shock to overcome this difficulties. They showed that introducing this shock smooths the value functions and, under monotonicity assumptions, it guarantees the existence of an equilibrium on discretized state spaces.

Appendix C   Data

This section describes the data sources and the procedures used to aggregate and combine data from different sources. The sample includes data for 18 emerging markets: Argentina, Brazil, Bulgaria, Chile, Colombia, India, Indonesia, Lithuania, Malaysia, Mexico, Peru, Philippines, Poland, Romania, Russia, South Africa, Turkey, and Ukraine [18]. It spans the period 1994-Q1 through 2016-Q1 with some significant missing periods for some countries.

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[18]The same countries were used in Aguiar et al. (2016b). They also include Latvia and Hungary, but due to data availability those could not be included in the regressions on table 2

GDP: Quarterly data on current and constant GDP at market prices, seasonally adjusted, from the Global Economic Monitor by the World Bank. The nominal data was used to calculate all the ratios (Current Account, Debt and Reserves ratio) while the real data was used to calculate the growth rate.

Spreads: Data for the monthly Emerging Market Bond Index (EMBI) of each country (blended spread), as well as the composite blended spread or EMBI+, was downloaded from Datastream. The values of March, June, September and December were used for the quarterly data.

International Reserves: Monthly Total International Reserves data from the Global Economic Monitor was used. The values of March, June, September and December were used for the quarterly data.

Current Account: The current account data corresponds to the total net US dollars current account from the World Economic Outlook dataset available from the International Monetary Fund.

Debt: Quarterly Public Sector External Debt data comes from different sources. The data for Argentina, Brazil, Chile, Colombia, India, Indonesia, Malaysia, Philippines, and South Africa was downloaded from Haver EMERGE database. All other countries except Ukraine were downloaded from the Quarterly Public Sector Debt database of the World Bank. Neither of these two previous sources provided enough data for Ukraine. To complement this country panel, annual data from the International Debt Statistics database from the World Bank was used. Linear interpolation was used to calculate the quarterly values.

Exchange rate regime: The exchange rate regime classification is taken from the monthly fine classification by Ilzetzki, Reinhart, and Rogoff (2011). This scale is then grouped into a dummy variable identifying those countries with a managed rate regime which includes: No separate legal tender, Pre-announced peg or currency board arrangement, Pre-announced horizontal band that is narrower than or equal to ±2%, De facto peg, Pre-announced crawling peg, Pre-announced crawling band that is narrower than or equal to ±2%, De facto crawling peg, De facto crawling band that is narrower than or equal to ±2%. Pre-announced crawling band that is wider than or equal to ±2%, De-facto crawling band that is narrower than or equal to ±5%, and Moving band that is narrower than or equal to ±2%.