IDB WORKING PAPER SERIES No. IDB-WP-440

Normative Fiscal Policy and Growth

Some Quantitative Implications for the Chilean Economy

Emilio Espino
Martín Gonzalez-Rozada

September 2013

Inter-American Development Bank
Fiscal and Municipal Management Division

Normative Fiscal Policy and Growth

Some Quantitative Implications for the Chilean Economy

Emilio Espino
Martín Gonzalez-Rozada

Universidad Torcuato Di Tella

Inter-American Development Bank
2013

Cataloging-in-Publication data provided by the
Inter-American Development Bank
Felipe Herrera Library

Espino, Emilio.
Normative fiscal policy and growth: some quantitative implications for the Chilean economy / Emilio Espino, Martin Gonzalez-Rozada.
p. cm. (IDB working paper series ; 440)
Includes bibliographical references.
1. Fiscal policy—Chile. 2. Economic development—Chile. 3. Tax administration and procedure—Chile. I. Gonzalez Rozada, Martín. II. Inter-American Development Bank. Fiscal and Municipal Management Division. III. Title. IV. Series.
IDB-WP-440

http://www.iadb.org

Normative Fiscal Policy and Growth: Some Quantitative Implications for the Chilean Economy

Emilio Espino
Universidad Torcuato Di Tella

Martin Gonzalez-Rozada
Universidad Torcuato Di Tella

October 24, 2013

Abstract[1]

This paper explores the qualitative and quantitative implications of optimal taxation in a developing economy when economic growth is endogenously determined. We differentiate this class of economies from a developed economy in two aspects: informal sector is quantitatively significant and tax-collecting technologies are more rudimentary. We characterize competitive equilibrium allocations and Ramsey allocations in the context of a small open economy in which the interest rate is endogenously determined, some workers can be hired in the informal market, and imperfect tax-collecting technology can be heterogeneous across different types of taxes. We calibrate the parameters of our model to the Chilean economy. Overall, our results suggest that capital should still be taxed but considerably less than actual taxes (that is, 10.78 percent versus 18.5 percent). Labor should be subsidized (to stimulate accumulation of human capital), while consumption taxes should be increased by 50 percent approximately (from 19 percent to 28 percent). As expected, the better the tax collecting technologies, the higher the corresponding taxes. In this context, the resulting growth rate increases only slightly along the balanced growth path.

JEL Classification: E61, E62, H21

Keywords: Optimal fiscal policy, economic growth, inefficient tax collecting technology.

[1] This publication was made possible thanks to the financial support of the Inter-American Development Bank (IDB). The authors are grateful for the helpful comments on earlier drafts of this paper by Ana Corbacho, Gustavo Garcia, Teresa Ter-Minassian, and all the participants at the "Fiscal Policy for Sustainable and More Equitable Growth in Latin America and the Caribbean" seminar, which took place in Washington, DC at the IDB in September 2012. They are also thankful to Andres Sarto, who provided excellent research assistance.

1 Introduction

In recent years, the determinants of the divergent paths of development across countries have deserved considerable attention. In fact, a body of research emphasizes that differences in development paths are closely related to differences in governments policies (see Jones, Manuelli and Rossi (1993)), giving rise to a fundamental question in macroeconomics: How should fiscal policy be optimally set over the long run?

This paper aims to add to the understanding of the connection between government policies and growth by laying out and extending recent developments within a framework that integrates tools from public finance to analyze macroeconomics and modern growth theory. Specifically, we explore quantitative assessments of the effect of drastic changes in the structure of fiscal policies relative to the current economy of a representative Latin American country: Chile. We examine the effect of setting optimal tax rates on the growth rate and other key variables in a representative-agent calibrated economy. This is, to the best of our knowledge, the first attempt to asses the quantitative implications of this type of fiscal reform, which can be a benchmark to discuss *any fiscal reform.*

We analyze optimal fiscal policy under a commitment to finance an exogenous path of public expenditures in a small open economy, and do so in the context of an endogenous growth model in which the degree of efficiency of the tax collecting technologies can vary across taxes and activities. We quantify the behavior of the economy along the competitive equilibrium balanced growth path (BGP) to understand how changes in taxes affect the variables in the long run. Then, using the characterization of the competitive equilibrium, we study the design of the optimal tax policy. To do so, we propose a model economy that includes some nonstandard assumptions to capture particular features of Latin American countries. In this economy, the labor market includes a formal sector and a less productive informal sector; the technology to collect taxes is neither perfect nor symmetric (i.e., one taxed unit does not necessarily transform into one unit of fiscal income [but possibly less], and different taxes can have different degrees of imperfection); and the domestic interest rate has an extra component determined by the level of domestic debt. We obtain solutions that are time inconsistent, a common characteristic of the proposed model. This is not unreasonable, since this is a normative analysis and the model does not aim to develop testable implications, but rather to provide quantitative guidelines for optimal decision making by governments. After designing the optimal path for the fiscal variables, the next step is to define the institutional environments that can support it.

The rest of the paper is organized as follows. The next section reviews the literature on optimal fiscal policy, and Section 3 describes the proposed model, a small open economy with endogenous growth, international capital mobility, and both formal and informal labor sectors.

Section 4 formalizes the competitive equilibrium and quantifies the behavior of the economy along the BGP. The findings suggests that introducing an informal sector into the economy and increasing labor, capital, and consumption taxes have a negative impact on the long-run growth rate. The latter effect is expected, since distortionary taxes should slow down the economy. Additionally and, again, as expected, increasing labor taxes leads to a reduction in the time devoted to work in the formal sector and an increase in the time allocated to work in the informal sector; however, the reduction is larger than the increase, resulting in a decline in total time allocated to work. Increasing capital taxes reduces the time devoted to work in both sectors. An increase in capital taxes also leads to more time spent in leisure activities and less in human capital accumulation. Increasing consumption taxes has similar effects; a rise in the consumption tax rate reduces not only the time devoted to work in both sectors, but also the time allocated to accumulate human capital. This implies an increase in the time devoted to nonmarket activities (i.e., the production of home goods), because agents can avoid this tax increment by consuming the untaxed good: leisure.

In Section 5, we study the behavior of this economy along the BGP when the government set taxes optimally. The goals are, first, to understand how the different tax collecting technologies affect optimal tax rates, growth, and time allocation, and second, to measure how much the tax rates observed in the Chilean economy should change to decentralize the Ramsey allocation problem (i.e., to switch to the optimal tax policy). The empirical evidence obtained indicates that the tax rates should change significantly. Section 6 provides specific policy recommendations, suggesting that capital–which, based on standard neoclassical growth models, should not be taxed–has to be taxed, albeit at a considerably lower rate than the rate observed in Chile (10.78 percent compared 18.5 percent, respectively). Labor should be subsidized (to stimulate accumulation of human capital), while consumption taxes should be increased by 50 percent (from 19 percent to 28 percent). In this context, the resulting growth rate increases only slightly along the BGP, despite significant changes in the time devoted to both the formal and informal labor markets, as well as to nonmarket activities. Section 7 concludes the paper.

2 Literature Review

There is a vast theoretical literature that studies optimal fiscal policy within the framework of some version of the neoclassical growth model.[2] Chamley (1986) shows that the long-run tax rate on capital should be zero. Lucas (1990) and Jones, Manuelli and Rossi (1993) extend this finding to an endogenous growth model. The basic intuition behind this result is that a capital income tax distorts the investment decision, so, in the long run, it should be replaced entirely by an income tax.

[2] A comprehensive survey of this area can be found in Chari, Christiano and Kehoe (1991).

This is an important result, since the optimal tax structure that it describes is significantly different from what it is observed in practice. As such, the model on which it is based requires further consideration. In particular, Correia (1996) study a situation in which the zero tax will not apply, analyzing a small open economy and assuming that there are one or more factors of production that the government cannot tax (or cannot tax optimally). Then, the tax on capital income will be dependent on the relationship between capital and the nontaxable factors. Our setting shares this extra ingredient. Given these theoretical results, actual tax systems are apparently far from these prescriptions, which raises the possibility that reforms in these systems can increase the growth rate and the welfare level. This suggests a purely quantitative question of whether one can justify a policy reform that considers a budget-balanced replacement of the capital tax by taxes on consumption or labor.

Lucas (1990) made the first major contribution in this respect in his analysis of an endogenous growth model with investment in human capital that drives growth in a representative agent setting. The model eliminates distributional issues to focus entirely upon efficiency. Using data from the U.S. economy, Lucas measures what would have happened if the tax on capital had been set to zero in 1985, with revenue neutrality ensured by increasing the tax on labor. With an initial capital tax rate of 36 percent, the rate of growth of output per capita before the tax reduction is 1.5 percent. In this setting, reducing the capital tax to zero causes a reduction in the growth rate to 1.47 percent, an increase of over 30 percent in the capital stock, and increases of 6 percent in consumption and 5.5 percent in welfare. Consequently, the policy change results in a significant level effect, but an insignificant growth effect. These findings can be explained as follows. Since time is the only input into the production of human capital, the cost (and return) is just the forgone wage. This leaves the human capital choice unaffected by taxation and, since it is this that drives growth, there is no growth effect. The level effect arises simply because of the replacement of a distortionary tax by a nondistortionary one.[3]

King and Rebelo (1990) extend the analysis of Lucas, considering both an open and closed economy. However, their model differs by having physical capital as an input into the production of human capital. In addition, King and Rebelo permit depreciation of both capital inputs. In their benchmark case, where the share of physical capital in human capital production is one-third, increases in the capital tax and labor tax from 20 to 30 percent reduce the growth rate by 1.52 percent (from 1.02 to –0.5). The level effect is a 62.7 percent decrease in welfare. A 10 percent increase in the capital tax alone reduces growth to 0.5 percent. When the share of physical capital

[3] Whereas Lucas considers only the differences between steady states, Laitner (1995) explicitly models the transition process. Along the transition process, there has to be an accumulation of physical capital, and hence a reduction in consumption, until the permanently higher level is achieved. Taking account of this will lower the increase in welfare. The results of Laitner suggest that taking full account of the transition will reduce the welfare gain by about 40 per cent, to give a net increase in welfare of 3.3 per cent.

in human capital production is decreased to 0.20, growth falls to 0.11 percent. In the open economy version of the model, which is characterized by an interest rate fixed at the global level, the fall in growth is even greater: a 10 percent increase in the capital tax reduces growth by 8.6 percent.

Jones, Manuelli and Rossi (1993) provide the most general and ambitious quantitative exercises, which combine elements from both Lucas and King and Rebelo; in particular, human capital requires time and goods to be produced. Jones et al. parameterize the utility function in a significantly different way than Lucas. Lucas' intertemporal marginal rate of substitution (IMRS) is 0.5 and the elasticity of labor supply (ELS) is 0.5. In contrast, Jones, Manuelli, and Rossi calibrate the ELS with the data and so, when IMRS is 0.5, the corresponding ELS is 4.99; for example, labor supply is much more elastic, implying, in turn, that taxation will have a greater distortionary effect. For $\sigma = 2$, Jones, Manuelli, and Rossi find that the elimination of all taxes (so distortions are completely removed) raises the growth rate from 2 to 5 percent, with a welfare gain of 15 percent (e.g., 1.15 is the factor by which the consumption path must be raised in order to bring utility under the current system up to the level attained in the Ramsey solution). For higher values of the IMRS, and hence greater values of ELS, the effect is even more dramatic.[4]

Summarizing these contributions, Lucas finds no growth effect, but a significant level effect. In contrast, King and Rebelo and Jones, Manuelli, and Rossi find very strong growth and level effects. King and Rebelo use a much lower share of human capital in its own production than Lucas and a depreciation rate of 10 percent. For human capital especially, this rate would seem excessive. For Jones, Manuelli, and Rossi, the higher degree of elasticity of labor supply leads to the divergence with Lucas.[5]

In this paper, we quantify the impact of implementing tax reforms that decentralize the optimal fiscal policy for the Chilean economy. The model economy proposed to study these issues, discussed in detail below, not only encompasses Jones, Manuelli and Rossi (1993) and Correia (1996), but also adds some elements that are key to studying Latin American and Caribbean (LAC) economies.

3 A Theoretical Framework

This section describes the physical setting, the asset market structure, and the government. As discussed above, some nonstandard assumptions are made to capture particular features of a prototype LAC economy.

[4] The reason for this increase in growth can be seen in the response of labor supply to the tax changes.

[5] The role played by each ingredient to explain the divergence between the results is studied in Stokey and Rebelo (1995), who use a model that encompasses the previous three.

3.1 Technology and Households

There is a neoclassical technology to produce a tradable consumption good in this prototype economy that displays constant returns to scale. Tradable goods are produced using effective units of labor and tradable capital. This technology is represented by

$$Y_t = F(K_t, L_t^F, L_t^I) = A \ (K_t)^{\alpha} \left(\beta \ \left(L_t^F\right)^{\frac{\eta-1}{\eta}} + (1-\beta) \ \left(L_t^I\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta(1-\alpha)}{\eta-1}},$$

where A is a technology parameter, $\alpha \in (0,1), \beta \in (0.5, 1]$ and $\eta \geq 0$. The distribution parameter β reflects intensity in units of effective formal labor, while α is the participation of capital. η is the elasticity of substitution between effective informal labor, $\left(L_t^I\right)$, and effective formal labor, $\left(L_t^F\right)$, while the elasticity of substitution between capital and composite labor is 1. When there is strict complementarity ($\eta < 1$), other things equal, a rise in L_t^F leads, in equilibrium, to an increase in informal labor. Conversely, when there is strict substitutability ($\eta > 1$), a rise in L_t^F induces a decrease in informal labor. If $\eta = 1$, this technology reduces the standard Cobb-Douglas production function.

Firms can hire workers either in the formal or the informal labor market. Informality translates into less productivity, ($\beta > 0.5$), and workers hired in the informal market do not pay labor taxes. At date t, firms pay wages w_t^F and w_t^I per unit of effective formal and informal labor, respectively.

Let C_t and x_t denote private consumption and leisure at date t, respectively. In this model, it is important to interpret leisure in a broad sense (i.e., including any nonmarket activities, such as home goods production). Representative household preferences are described by time-separable, discounted utility, where $\{C_t, x_t\}_{t=0}^{\infty}$ is valuated

$$\sum_{t=0}^{\infty} \rho^t \ \frac{\left(C_t \ (x_t)^{\theta}\right)^{1-\sigma}}{1-\sigma},$$

where $\rho \in (0,1)$, $\sigma > 0$, and $\theta \geq 0$. ρ is the discount rate and σ is the intertemporal elasticity of substitution. It is worth mentioning that, as also proposed by Lucas (1990), if agents do not value leisure ($\theta = 0$), then taxes have no impact on growth rates.

The representative household is endowed with a unit of time every period, which must be allocated across three types of activities and leisure; that is, effective units of labor are given by

$$\begin{aligned} L_t^F &= u_t \, H_t \\ L_t^I &= v_t \, H_t, \end{aligned}$$

where u_t and v_t is the date$-t$ fraction of time working in the formal and informal sector, respectively and H_t is the date$-t$ stock of human capital that evolves according to

$$H_{t+1} = A^H \left(1 - u_t - v_t - x_t\right) H_t + \left(1 - \delta_H\right) H,$$

where $A^H > 0$ is a human capital technology parameter and $\delta_H \in (0, 1)$ denotes the human capital depreciation rate. $(1 - u_t - v_t - x_t)\ H_t$ is interpreted as the effective units of labor allocated in the human capital sector at date t.

3.2 Factor Mobility, Asset Market Structure, and the Government

Let G_t denote public consumption expenditures at date t. We consider a benevolent government that provides public goods, G_t; levies linear taxes on labor, capital, and consumption; and issues debt. We assume that $G_t = gY_t$ for all t; that is, $g \in (0, 1)$ is the government spending to income ratio.

The government can levy a tax of $\tau_K \in [0, \overline{\tau}_K]$ on the the net return on capital, $(r_t - \delta_K)$ K_t, where r_t denotes the domestic rental price of capital before taxes.[6] Think of τ_K as a tax on corporate profits that is levied on firms operating in the country. The government can also tax consumption at the rate τ_C and the formal sector at the rate τ_w. As stressed above, the informal sector does not pay taxes.

Tax collecting technologies are neither perfect nor symmetric. The first feature means that one unit taxed does not necessarily transform into one unit of fiscal income (but possibly less). The second feature means that different taxes can have different degrees of imperfection. Each unit of capital, labor, and consumption taxes transforms in e_k, e_w and $e_c \in (0, 1)$ units of fiscal proceeds, respectively.

There is a one-period bond to trade internationally at the price $q_t = 1/R_t$, where R_t denotes the gross interest rate, which will be determined endogenously. The government and households have access to the credit market. Let B_t^p and B_t^g denote private and government asset holdings, respectively, and $B_t = B_t^p + B_t^g$. The government's budget constraint is

$$B_{t+1}^g + G_t = e_c\,\tau_t^C\,C_t + e_w\,\tau_t^W\,w_t^F\,L_t^F + \,e_k\,\tau_t^K\,\left(r_t - \delta_K\right)\,K_t + \left(1 + R_t\right)\,B_t^g,$$

where $\left\{B_{t+1}^g\right\}_{t=0}^{\infty}$ is further restricted by a no-Ponzi condition, which will be specified later. We denote $\pi = \left\{\tau_C, \tau_W, \tau_K, G_t, B_{t+1}^g\right\}_{t=0}^{\infty}$ as a *fiscal policy.*

There is no international labor mobility, and physical capital is restricted as follows. It is necessary to invest domestically to produce new capital. Let K_t denote the domestic stock of

[6] Following the convention in the literature we assume that return on capital after depreciation are taxed.

capital at date t in units of consumption goods. The law of motion for capital is given by

$$K_{t+1} = I_t + (1 - \delta_K)\, K_t,$$

where I_t denotes domestic investment at date t and $\delta_K \in (0, 1)$ denotes the depreciation rate. Notice that agents can borrow one unit in the bond market at date t to invest domestically and produce one unit of capital at $t + 1$.

The domestic interest rate depends negatively on the domestic asset (debt) to capital ratio

$$R_t = R\left(\frac{B_t}{K_t}\right),$$

where $R' < 0$. Domestic agents take this rate as given; that is, they do not internalize the impact of alternative debt choices.

The market clearing condition for the labor market reduces to

$$\begin{aligned} L_t^F &= u_t\, H_t \\ L_t^I &= v_t\, H_t \end{aligned}$$

for all t.

4 Competitive Equilibrium Analysis

In this section, we formalize the corresponding competitive equilibrium concept (Subsection 4.1), and then we quantify the behavior of the economy along the BGP (Subsection 4.2). The goal of this section is twofold. First, we find useful to understand how changes in taxes affect the variables in the long run. Second, we use the characterization of the competitive equilibrium to study the design of the optimal tax policy in this context by means of the primal approach discussed in Section 5.

4.1 Fiscal Policy and Competitive Equilibrium

Given a fiscal policy $\pi = \left\{\tau_C, \tau_W, \tau_K, G_t, B_{t+1}^g\right\}_{t=0}^{\infty}$ and prices $\left\{R_t, w_t^F, w_t^I\,, r_t\right\}_{t=0}^{\infty}$, the representative household solves

$$\max_{\{C_t, u_t, v_t, K_{t+1}, H_{t+1}, B_{t+1}^p\}} \sum_{t=0}^{\infty} \rho^t\, \frac{\left(C_t\ (x_t)^{\theta}\right)^{1-\sigma}}{1-\sigma},$$

subject to

$$
\begin{aligned}
& (1+\tau_C)C_t + (K_{t+1} - K_t) + B^p_{t+1} \\
= \;& (1-\tau_W)\, w^F_t\ u_t\ H_t + w^I_t\ v_t\ H_t + (1-\tau_K)\ \ (r_t - \delta_K)\ \ K_t + (1+R_t)\, B^p_t
\end{aligned} \tag{1}
$$

$$
H_{t+1} = A^H\ (1 - u_t - v_t - x_t)\ H_t + (1-\delta_H)\ H_t \tag{2}
$$

$$
\lim_{T\to\infty} \prod_{j=0}^{T} \frac{B^p_T}{(1+R_j)} \geq 0,
$$

where (K_0, H_0, B_0) are given (the last condition restricts $\left\{B^g_{t+1}\right\}_{t=0}^{\infty}$ to rule out Ponzi schemes). The households pay for the consumption tax, as well as labor and capital taxes. In equilibrium, it is indistinct if either the firms or workers pay the factor taxes.

Given π, denote $\{C_t(\pi), u_t(\pi), v_t(\pi), K_{t+1}(\pi), H_{t+1}(\pi), B^p_{t+1}(\pi)\}_{t=0}^{\infty}$ and $\left\{R_t(\pi), w^F_t(\pi), w^I_t(\pi)\ , r_t(\pi)\right\}_{t=0}^{\infty}$ as the corresponding solution to the representative household's problem and prices, respectively. We say that a fiscal policy is *feasible* if

$$
\begin{aligned}
B^g_{t+1} + G_t \;\; = \;\; & e_c\, \tau_C\ C_t(\pi) + e_w\, \tau_W\ w^F_t(\pi)\ u_t(\pi)\ H_t(\pi) \\
& + e_k\, \tau_K\ \ (r_t(\pi) - \delta_K)\ \ K_t(\pi) + (1 + R_t(\pi))\ \ B^g_t
\end{aligned}
$$

for all t; that is, a fiscal policy is feasible if it satisfies the government budget constraint. We restrict ourselves to feasible fiscal policies without any further reference.

Definition 1. *A competitive equilibrium (CE) is an allocation* $\{C_t, x_t, u_t, v_t, K_{t+1}, H_{t+1}, B^p_{t+1}\}_{t=0}^{\infty}$ *and a price system* $\left\{R_t, w^F_t, w^I_t\ , r_t\right\}_{t=0}^{\infty}$*, such that, given a fiscal policy* $\pi = \left\{\tau_C, \tau_W, \tau_K, G_t, B^g_{t+1}\right\}_{t=0}^{\infty}$*, the following conditions are satisfied:*

CE.1. Given $\left\{R_t, w^F_t, w^I_t\ , r_t\right\}_{t=0}^{\infty}$*, the allocation* $\{C_t, u_t, v_t, K_{t+1}, H_{t+1}, B^p_{t+1}\}_{t=0}^{\infty}$ *solves the representative household's problem.*

CE.2. Fiscal policy $\pi = \left\{\tau_C, \tau_W, \tau_K, G_t, B^g_{t+1}\right\}_{t=0}^{\infty}$ *is feasible.*

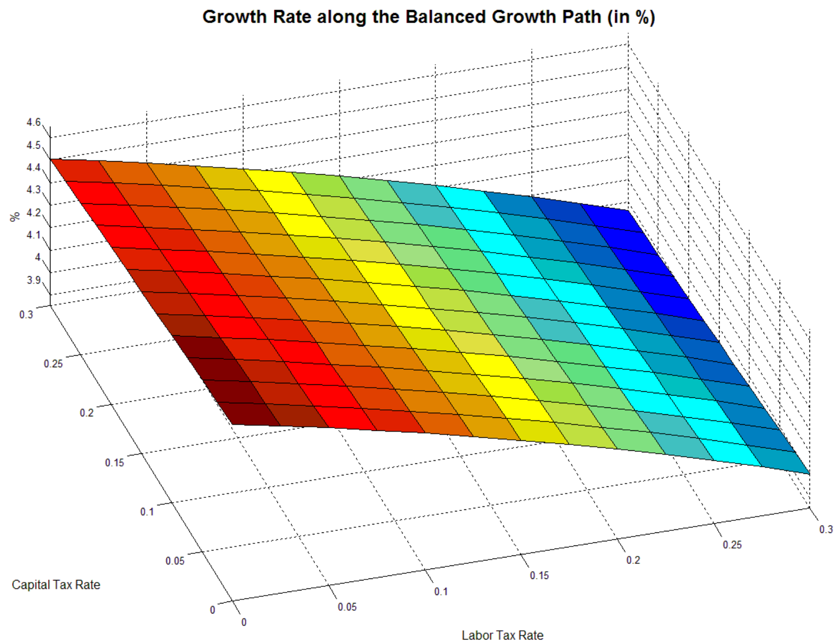
CE.3. Firms maximize static profits; that is, for all t

$$
\begin{aligned}
r_t &= F_1(K_t, u_tH_t, v_tH_t) \\
w^F_t &= F_2(K_t, u_tH_t, v_tH_t) \\
w^I_t &= F_3(K_t, u_tH_t, v_tH_t).
\end{aligned}
$$

CE.3. There is consistency of the domestic interest rate; that is, for all t

$$
R_t = R\left(B_t/K_t\right).
$$

Figure 1. Growth Rate along the BGP, Formal Sector Only (in percent)



Source: Authors' estimations.

4.2 Balanced Growth Analysis: Competitive Equilibrium

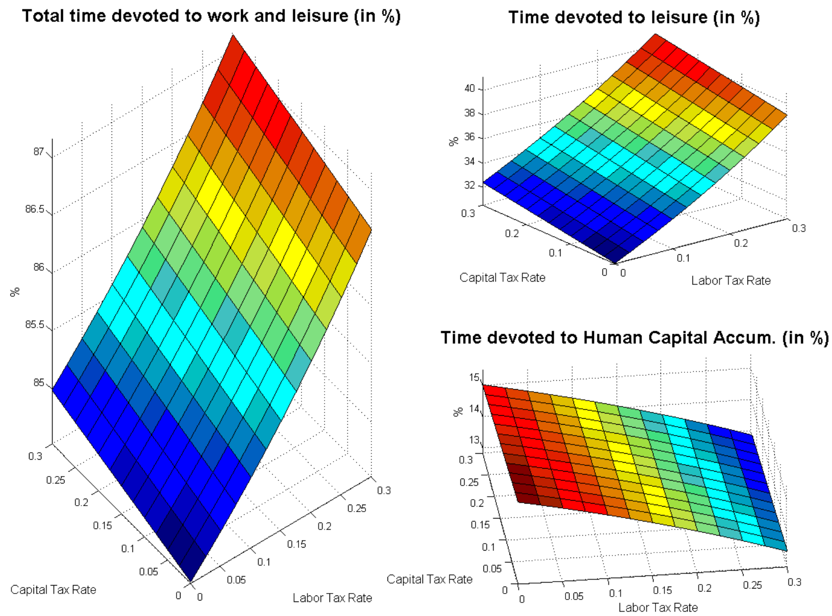
We are particularly interested in studying the BGP. In the Appendix, we characterize a competitive equilibrium for this economy and its corresponding BGP. The model displays some features that are not standard for a small open economy. First, the growth rate is endogenously determined by the fact that the interest rate depends on a measure of relative indebtedness. This friction will be critical to close the model for a developing economy such as Chile's. Second, imperfect tax collecting technologies appear only in the aggregate restrictions, that is to say, in the budget constraint of the government and the aggregate budget constraint of the open economy. Given this feature, we fix $e_w = e_\tau = e_c = 1$ and change the tax rates.

4.2.1 ***Only Formal Sector***

The first exercise consists of removing the informal sector of the economy $(\beta = 1)$ to see how τ_K and τ_w affect the BGP. Figure 1 shows how capital and labor taxes affect the economy's growth rate in this scenario.

As Figure 1 illustrates, increasing capital and labor tax rates reduces the growth rate along the BGP. However, changes in the labor tax rate have a greater effect over the growth rate than

Figure 2. Time Devoted to Work, Leisure and Human Capital Accumulation, Formal Sector Only

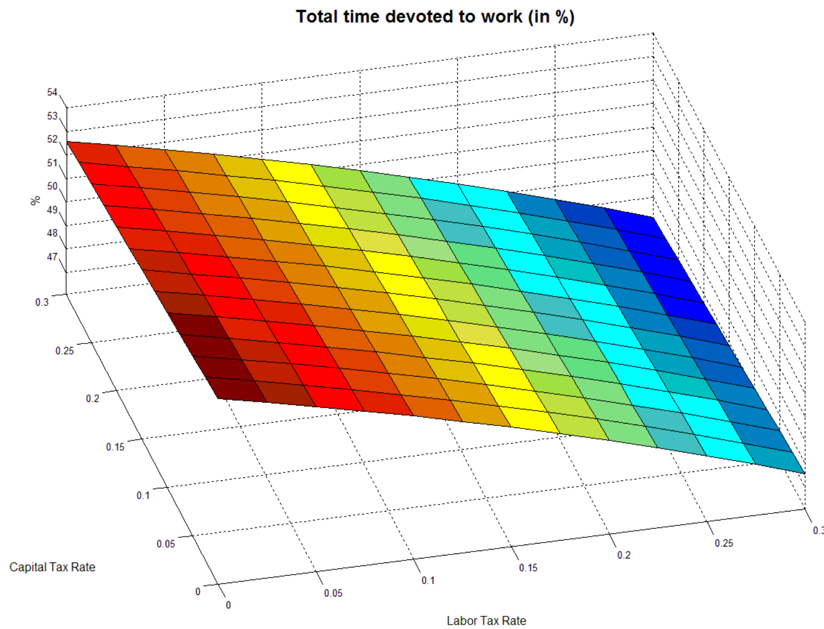


Source: Authors' estimations.

changes in the capital tax rate. Increasing the labor tax rate by 30 percent reduces growth by more than 8 percent, while the same increase in the capital tax rate reduces the growth rate by around 3 percent. This is expected, since distortionary tax rates should slow down the economy.

Figure 2 shows how changes in capital and labor taxes affect time allocation between leisure and human capital accumulation. The panel on the left in the figure shows the effect on the total time devoted to work and leisure, while the panel on the right shows the effects on the time devoted to leisure and human capital accumulation. The results show that increasing the labor tax rate has a positive effect on the time devoted to leisure, as expected, and a negative effect on the time devoted to human capital accumulation. Increasing the capital tax rate has an imperceptible effect on the time devoted to accumulate human capital and a positive effect on the time allocated to leisure. Raising the capital tax rate by 30 percent leads to an increase of 6 percent in the time devoted to nonmarket activities (i.e., nonmarket production goods consumed by the agents). It is important to interpret leisure in a broad sense, as modeled, and thus it must include nonmarket production goods. In other words, an agent that devotes less time accumulating human capital is not necessarily at home doing nothing; rather, he or she could be engaged in nonmarket activities (i.e., producing goods). Still, a large increase in those activities deserves further study.

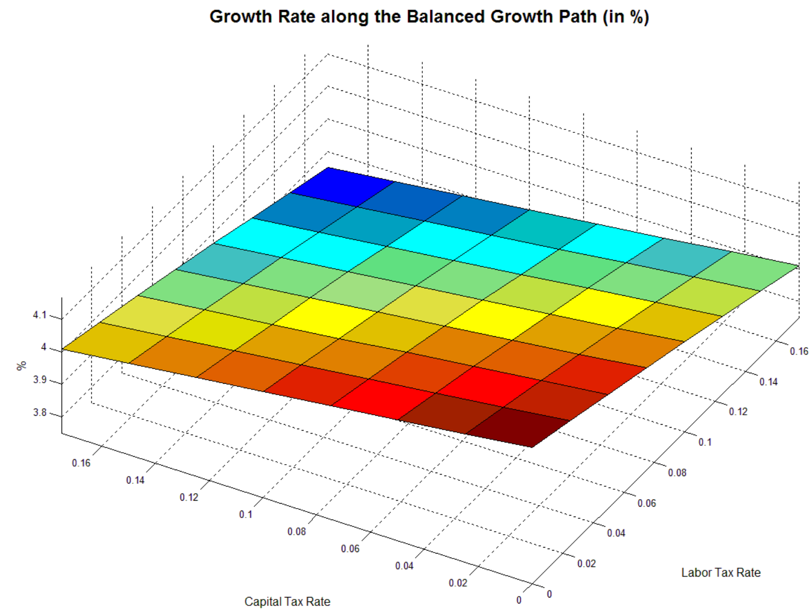
Figure 3. Total Time Devoted to Work, Formal Sector Only (in percent)



Source: Authors' estimations.

Overall, an increase in both tax rates has a positive effect on the time devoted to work in the formal sector of the economy and leisure (nonmarket activities). Figure 3 disaggregates this last effect, showing how taxes affect the time devoted to work. As labor taxes increase, the time devoted to work decreases, while an increase in the capital tax rate of 30 percent reduces the time devoted to work by 2.5 percent. In sum, from both figures, it seems that increasing the capital tax rate has a larger effect on the time devoted to nonmarket activities than its effects on the time devoted to work. Increasing the labor tax rate by 30 percent has a positive impact on the time allocated to leisure (17 percent increase), but a negative impact on the time devoted to both work and accumulating human capital (7 percent decrease). In other words, the effect of an increase in both tax rates observed in the left panel of Figure 2 is due to the impact on leisure.

Figure 4. Growth Rate along the BGP, Formal and Informal Sectors



Source: Authors' estimations.

4.2.2 Formal and Informal Sector

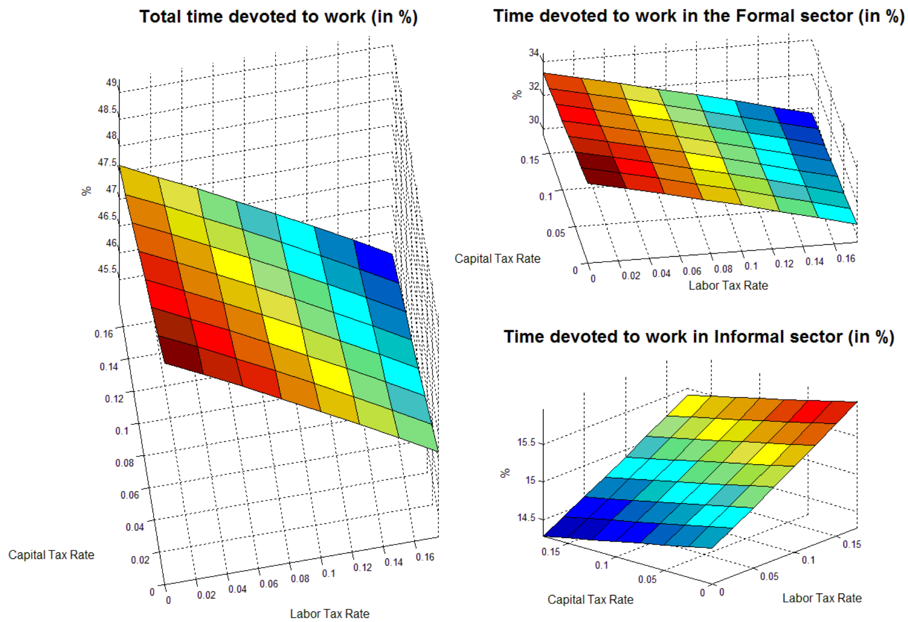
The second exercise introduces the informal sector into the economy. Here, we extend our analysis to study the impact of capital and labor taxes as well as the impact of capital and consumption taxes.

The Impacts of Capital and Labor Tax Rates

Figure 4 shows the impact of changes in the capital and labor tax rates on the growth rate along the BGP in an economy with both formal and informal sectors. As in an economy without an informal sector, an increase in both tax rates has a negative effect on the growth rate. Overall, the average growth rate in an economy with an informal sector is lower than the average growth rate without one. A comparison of Figures 1 and 4 suggests that the introduction of an informal sector into the economy diminishes growth; in particular, even if there is no change in the tax rates, the growth rate decreases from 4.6 to 4.2 percent.

As Figure 5 illustrates, as the labor tax rate increases, time devoted to working in the formal sector decreases, while time devoted to working in the informal sector increases, which are expected results of this model. However, the figure also suggests that the total time devoted to work decreases with this tax rate, implying that the disincentive to work in the formal sector is, on

Figure 5. Time Devoted to Work, Formal and Informal Sectors (in percent)



Source: Authors' estimations.

average, greater than the incentive to work in the informal sector. Increasing the capital tax rate reduces the time devoted to work in both sectors.

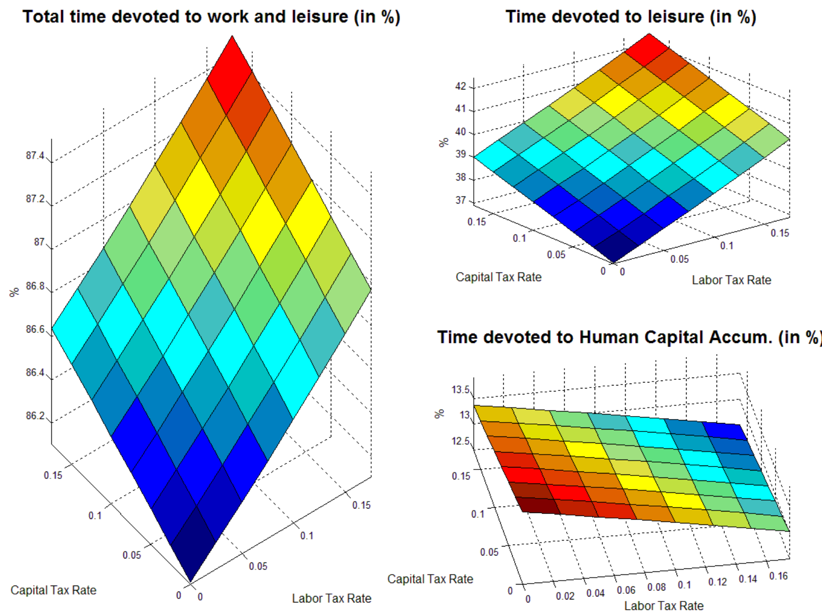
In the same line, Figure 6 shows the effects of capital and labor tax rates on the amount of time devoted to leisure and human capital accumulation. In both cases, a rise in the tax rate leads to an increase in the time allocated to nonmarket activities and a decrease in the time devoted to human capital accumulation.

The Impacts of Capital and Consumption Taxes

Figure 7 shows the impacts of capital and consumption taxes on the growth rate along the BGP. Both taxes negatively affect the long-run growth rate, due to additional distortions. When both taxes are zero, the growth rate is 4.2 percent. If both tax rates are increased to 10 percent, the growth is only 3.9 percent. Again, this is expected, since distortionary taxes tend to slow down the economy.

In terms of the impact on time allocation, Figure 8 shows that the time devoted to work in both the formal and informal sectors decreases with both tax rates, as does the total time devoted to work.

Figure 6. Time Devoted to Work, Leisure and Human Capital Accumulation, Formal and Informal Sectors (in percent)



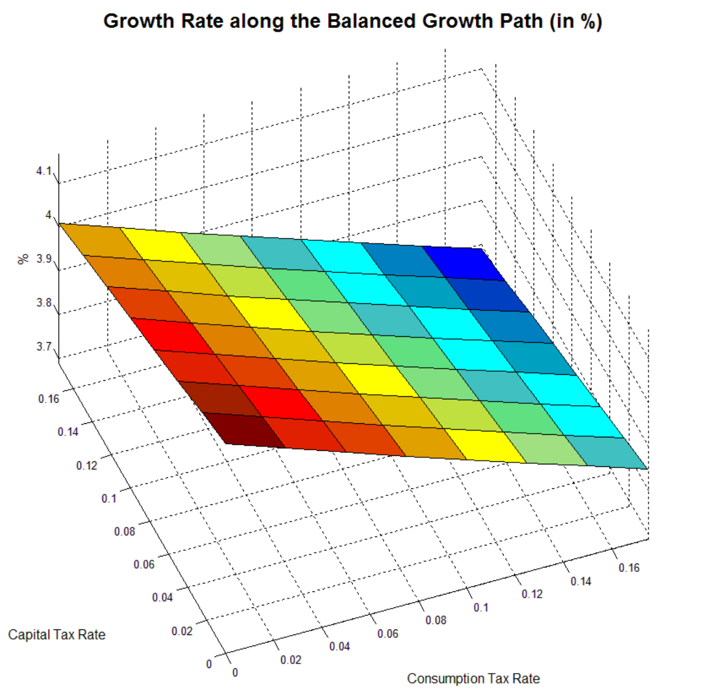
Source: Authors' estimations.

Similarly, Figure 9 illustrates that the time devoted to both work and human capital accumulation decreases with both tax rates. This implies that, at the same time, the time allocated to leisure increases significantly. In regards to consumption, an increase in the tax rate has a negative impact on long-run growth, because agents avoid this tax increment by consuming more heavily the untaxed good: leisure. This impact will decrease both labor market participation and time devoted to accumulate human capital.

Overall, the evidence found in this section suggests that the introduction of an informal sector into the economy and increasing labor, capital, and consumption taxes have a negative impact on the long-run growth rate. This last effect is expected, since distortionary taxes should slow down the economy. Additionally, and again as expected, an increase in labor taxes reduces the time devoted to work in the formal sector and increases the time devoted to work in the informal sector. However, the reduction in the formal sector is greater than the increase in the informal sector, resulting in a decline in total time allocated to work.

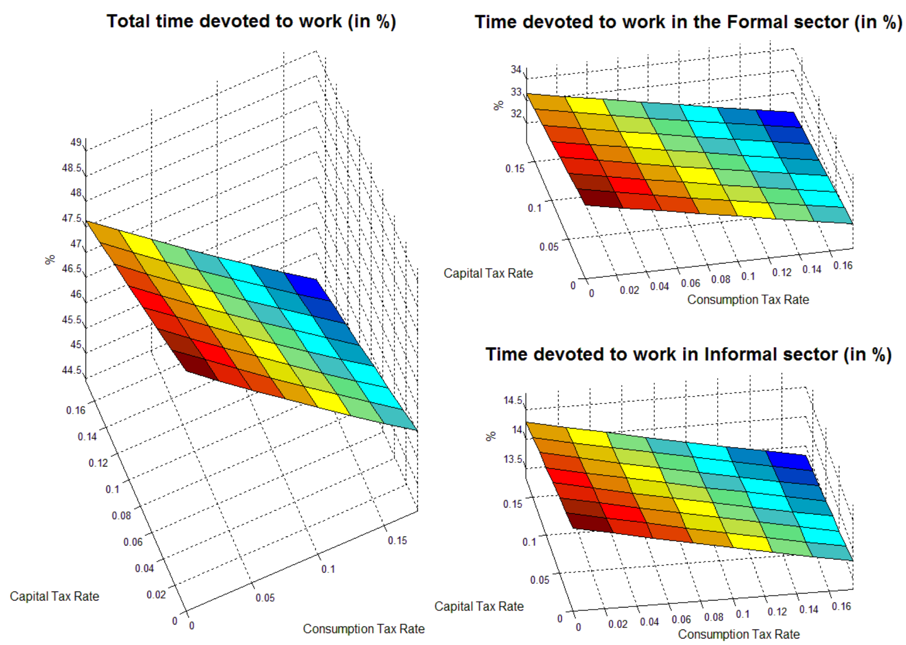
An increase in capital taxes leads to a decrease in the time devoted to work both in the formal and informal sectors. Also, as capital tax rates rise, time devoted to leisure increases and time devoted to human capital accumulation decreases. An increase in consumption taxes has a similar effect. A rise in consumption tax rates reduces not only the time devoted to work in both

Figure 7. Impact of Capital and Consumption Taxes on the Growth Rate along the BGP (in percent)



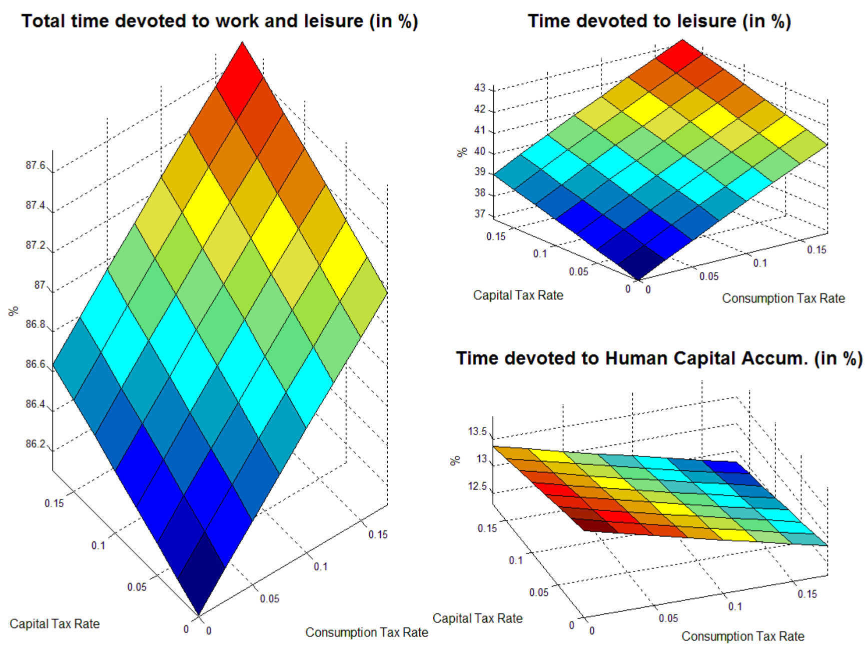
Source: Authors' estimations.

Figure 8. Impact of Capital and Consumption Taxes on Time Devoted to Work, Formal and Informal Sectors



Source: Authors' estimations.

Figure 9. Time Devoted to Work, Leisure and Human Capital Accumulation, Formal and Informal Sectors (in percent)



Source: Authors' estimations.

sectors, but also the time allocated to accumulate human capital. This implies an increase in the time devoted to nonmarket activities (i.e., home goods production), because agents can avoid these increments by consuming the untaxed good: leisure.

In the next section, we study the behavior of this type of economy along the BGP when the government sets optimal tax rates. The goals are, first, to understand how the different tax collecting technologies affect optimal tax rates, growth, and time allocation, and second, to measure how much the tax rates observed in the Chilean economy should change to decentralize the Ramsey allocation problem (i.e., to switch to the optimal tax policy). Once this last question is answered, the discussion moves to the implications these policies have on the growth rate and the allocation of time along the BGP.

5 Dynamic Optimal Taxation: The Ramsey Problem

In this section, we study a dynamic optimal taxation problem called a Ramsey problem with a solution called a Ramsey plan. The government's goal is to maximize households' welfare subject to raising set revenues through distortionary taxation. When designing an optimal policy, the government takes into account the equilibrium reactions by consumers and firms to the tax system.

The nature of efficient taxation arises out of the tension between two principles, both of which are familiar from Ramsey's original static analysis. One principle is that factors of production in inelastic supply–factors whose income is pure rent–should be taxed at confiscatory rates. In the present application, for instance, if consumers' initial capital holdings can be taxed directly via a capital levy, this eases the government constraint and reduces (or possibly eliminates entirely) the need to resort to distorting taxes. In the same way, defaulting on initial government debt and reducing promised transfer payments from government to households will reduce the need to resort to distorting taxes and improve welfare. Insofar as the government's ability to obtain capital levies in this general sense is left unrestricted, it will allow for full use of these tax sources. The present analysis assumes away capital levy possibilities.

A second principle in Ramsey's analysis is that goods that appear symmetrically in consumer preferences should be taxed at the same rate (i.e., taxes should be spread evenly over similar goods). In our application, this principle means that taxes should be spread evenly over consumption at different dates. Since capital taxation applied to new investment involves taxing later consumption at heavier rates than early consumption, this second principle implies that it is not a good idea to tax capital. In our formulation, there is only one tax rate applied to income from old and new capital alike, so these two principles cannot simultaneously be obeyed.

In order to study this taxation problem, we first formalize the so-called *primal approach*. Then, we quantify the behavior of the economy along the BGP in the case where the implicit taxes are set optimally.

5.1 The Primal Approach

Our approach builds on the primal approach to optimal taxation [see, for example, Atkinson and Stiglitz (1972), Lucas and Stokey (1983), and Chari, Christiano and Kehoe (1991)]. This approach characterizes the set of allocations that can be implemented as a competitive equilibrium with the distortion of taxes by two sets of conditions: resource constraints and implementability constraints. The latter refers to the consumer budget constraints, in which the consumer and the firms' first-order conditions are used to substitute out for prices and taxes. Thus, both constraints depend only on allocations, which implies that optimal allocations are solutions to a programming problem. The basic idea is to recast the problem of choosing optimal taxes as one of choosing allocations that are subject to constraints regarding which types can be supported as a competitive equilibrium for some taxes.

The Appendix provides details of how to solve the Ramsey problem applying the primal approach. Without loss of generality, we normalize $e_c = 1$ (that is, we assume the tax collecting technology for the consumption tax in Chile is more efficient than those of the other two taxes), and denote the wedges that represent labor and capital taxes, respectively, as follows:

$$\begin{aligned} taow_t &\equiv \left(\frac{F_2(t) - F_3(t)}{F_2(t)}\right) = 1 - \frac{\beta}{\alpha}\frac{u_t}{v_t} \\ taor_t &\equiv \left(\frac{F_1(t) - \delta_K - R_t}{F_1(t) - \delta_K}\right) = \left(1 - \frac{R_t}{(1-\alpha-\beta)\, A\, (k_t)^{-\alpha-\beta}\, (u_t)^{\alpha}\, (v_t)^{\beta} - \delta_K}\right), \end{aligned}$$

where $F_j(t)$ stands for the partial derivative of F with respect to the j-th argument and $k_t = K_t/H_t$.

Let ϕ be the Lagrange multiplier corresponding to the incentive constraint (22) in the Appendix and define

$$V(C_t;\phi) \equiv U(C_t, x_t) + \phi\, U_C(C_t, x_t)\, C_t,$$

where U_C denotes the partial derivative of U with respect to consumption.

The Ramsey problem for this economy reduces to

$$\max_{(C_t, u_t, v_t K_{t+1}, H_{t+1}, B_{t+1})} \sum_{t=0}^{\infty} \rho^t\, V(C_t;\phi) \tag{3}$$

subject to

$$U_C(C_t, x_t) = \rho\, U_C(C_{t+1}, x_{t+1})\, F_3(t+1)\ \left[A^H(1 - x_{t+1}) + (1 - \delta_H)\right] \tag{4}$$

$$H_{t+1} = A^H\,(1 - u_t - v_t - x_t)\, H_t + (1 - \delta_H) H_t \tag{5}$$

$$\begin{aligned} & C_t + (K_{t+1} - K_t) + B_{t+1} + g\, Y_t \\ = \; & [1 - taow_t\ (1 - e_w)]\ F_2(t)\, u_t\, H_t + F_3(t)\, v_t\, H_t \\ & + [1 - taor_t\ (1 - e_k)]\ (F_1(t) - \delta_K)\ K_t + (1 + R_t)\ B_t. \end{aligned} \tag{6}$$

The objective function (3) stems from coupling the utility function and the implementability constraint. This last constraint should be considered an infinite-horizon version of the budget constraint of either the consumer or the government, where the consumer and firm first-order conditions have been used to substitute out the prices and taxes. Next, restriction (4) captures the idea that, when allocating resources, the planner must take into account that agents choose optimal taxes intertemporally. Restriction (6) represents the period-by-period resource constraint in a small open economy. This constraint is adapted to accommodate alternative tax collecting technologies, which implies that there are additional wedges that the planner needs to optimally manipulate. In particular, $taow_t$ and $taor_t$ disappear if $e_w = e_k = 1$ (i.e., if tax collecting technologies are perfect). In the Appendix, we show how to characterize a Ramsey allocation in this setting with the nonstandard features.

5.2 Balanced Growth Analysis: Ramsey Allocation

The Appendix provides details on the conditions that characterize a Ramsey allocation along the BGP. Imperfect tax collecting technologies are important features that distinguish our setting from those in existing literature. In particular, these technologies make it clear that the limiting capital tax (as well as the labor tax) will not necessarily equal zero. To clarify, the steady state optimal taxes on capital and labor are given by

$$\begin{aligned} taor^* &\equiv 1 - \frac{R(b^*/k^*)}{(1 - \alpha - \beta)\, A\, (k^*)^{-(\alpha+\beta)}\, (u^*)^{\alpha}\, (v^*)^{\beta} - \delta_K} \\ taow^* &\equiv 1 - \frac{\beta}{\alpha}\frac{u^*}{v^*}, \end{aligned}$$

where "*" variables denote their levels along the BGP.

Note that $taor^*$ is equal to zero along the BGP only when the net return on capital equals the gross interest rate. In this setting, the government might choose to optimally distort that margin, given the effects of different tax collecting technologies. On the other hand, if $taow^*$ equals zero in equilibrium, both sectors are treated equally; that is, there is no informal sector, since informality here implies that both the firms and the workers avoid paying labor taxes. The government might optimally choose to distort this margin as well.

The balanced growth rate is determined by

$$\gamma^* = A^H (1 - u^* - v^* - x^*) + (1 - \delta_H),$$

so, the higher the amount of time devoted to accumulate human capital, $(1 - u^* - v^* - x^*)$, the higher the growth rate. Importantly, an economy's growth rate does not depend on taxes if agents do not value leisure (see Lucas (1990) for a similar result).

The tax collecting technology parameters for Chile are calibrated to

$$e_K = 0.69 \text{ and } e_W = 0.82,$$

where e_C is normalized to 1 (see Jorrat (2012)) in order to set the tax collecting technology for consumption taxes as the most efficient of the three technologies. This calibration implies that the efficiency of the technology for capital taxes is around 70 percent of the efficiency of the technology for consumption taxes, while for labor taxes, it is close to 80 percent. In the following exercises, we analyze the behavior and predictions of the Ramsey allocation along the BGP, which will illustrate the impact of changing the tax collecting technologies around these levels.

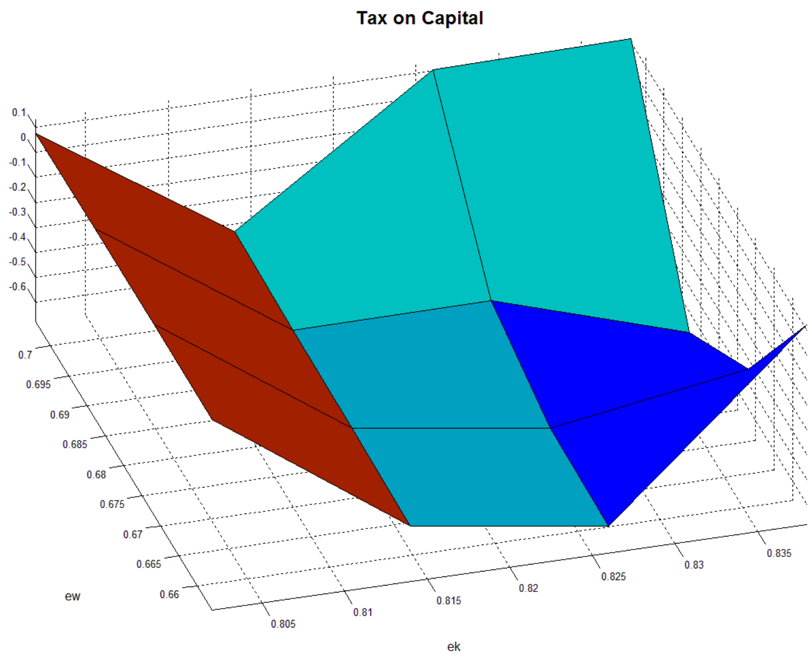
Figure 10 illustrates the effect of the optimal tax on capital as a function of the tax collecting efficiency parameters, $e_w \in (0.80, 0.84)$ and $e_k \in (0.66, 0.70)$. These parameters introduce novel effects on the choice of optimal variables (and thus of optimal taxes) that are absent in the setting with perfect tax collecting technologies.

The most important observation is that the relationship is non-monotonic. One would speculate that the higher the efficiency level of the tax collecting technology for capital, the higher the optimal capital tax. However, due to several interacting effects, this is not always the case. Depending on (e_w, e_k), optimal limiting taxes on capital can be either positive or negative. Zero capital taxes are rarely optimal.

Similarly, optimal labor and consumption taxes vary significantly, and they can be either positive or negative, depending on the tax collecting technology (see Figure 11). It is important to notice that the impact on taxes is larger for e_k than for e_w. Our intuition is that e_k impacts capital accumulation directly, while e_w impacts human capital accumulation indirectly.

In regards to the formal and informal labor markets, and the corresponding optimal growth rate along the BGP, again the relationship is non-monotonic, and the impact on labor market participation is larger for e_k than for e_w (see Figure 12).

Figure 10. Tax on Capital

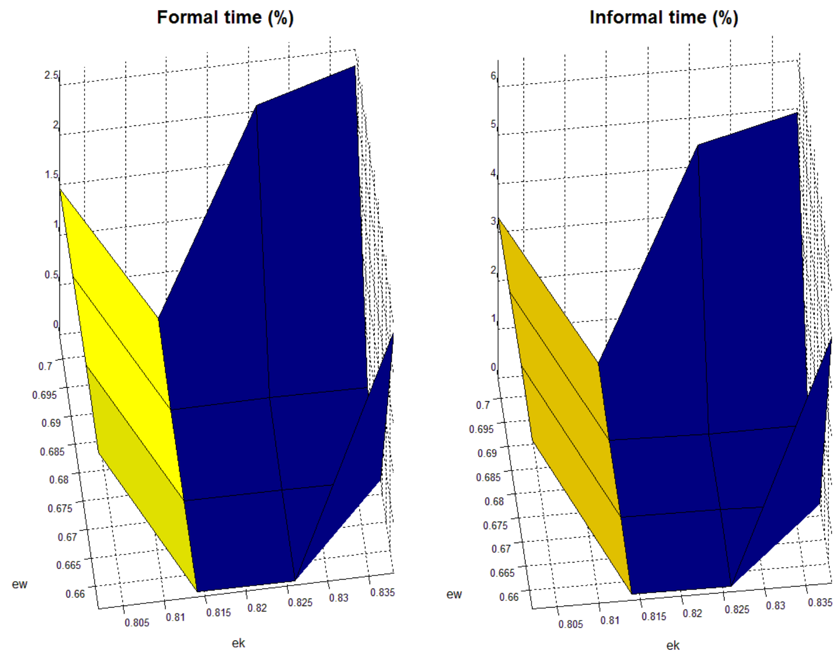


Source: Authors' estimations.

Figure 11. Tax on Wages and Consumption

Source: Authors' estimations.

Figure 12. Formal and Informal Time



Source: Authors' estimations.

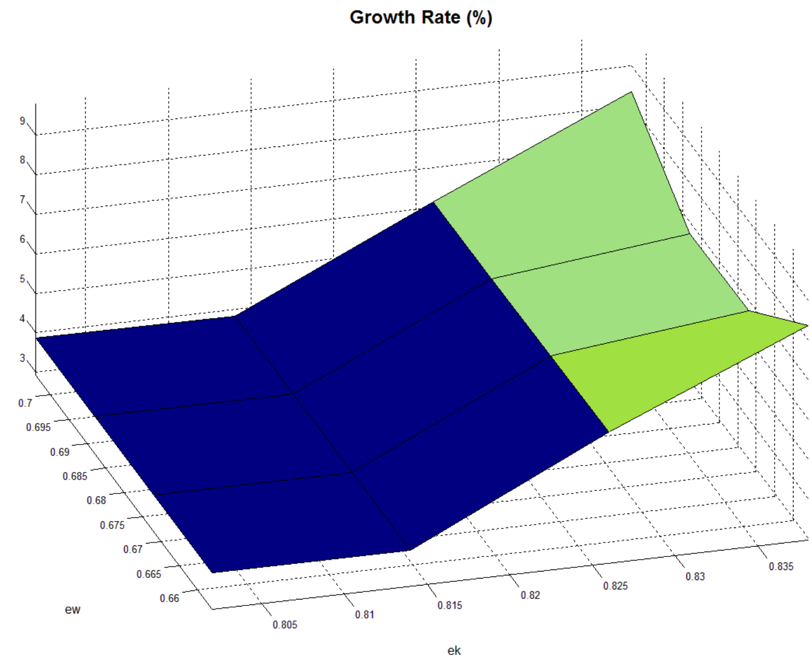
Growth rates behave non-monotonically; however, in general, better technology leads to increased growth (see Figure 13). This peculiar behavior needs further analysis; however, for the present study it is possible to grasp the concept by examining the conditions that characterize the Ramsey solution (see Appendix 2). When $e_W = 1, e_K = 1$, several effects shut down. On the other hand, when $e_W \neq 1$ and/or $e_K \neq 1$, the effects become active, and further analysis is needed to identify the consequences.

6 Policy Implications

This section provides some policy implications of our quantitative exercises. We aim to answer the following two questions:

1. What are the changes in the tax system needed to implement the Ramsey allocation?
2. What are the effects of these changes on the growth rate and the allocation of time along the BGP?

Figure 13. Growth Rate



Source: Authors' estimations.

Throughout this exercise, we keep the tax collecting technologies where the coefficients are calibrated to (see Jorrat (2012))

$$e_K = 0.69\,, \quad e_W = 0.82.$$

Table 1 summarizes our quantitative findings and exposes three facts along the BGP. First, comparing optimal tax rates to those observed in the Chilean economy, the tax rate changes needed to decentralize the Ramsey allocation are significant. First, capital should be taxed, but at a decreased rate, as the optimal rate is lower than the observed rate along the BGP (10.78 percent and 18.5 percent, respectively), while formal labor should be heavily subsidized, based on the observed and optimal rates (2 percent and -9.2 percent, respectively). On the other hand, consumption should be more heavily taxed, based on the observed and optimal rates (19 percent and 28.06 percent, respectively). Some findings are important to mention. First, unlike conclusions in previous studies, capital should be taxed in the long run and, as a matter of fact, at a relatively high rate. Labor taxation seems a bad idea in this setting, because, among other reasons, it encourages workers to move from the formal to the informal sector of the economy. Finally, consumption tax should be used more intensively, since it is less harmful in terms of distortions. Also, as expected, efficiency dictates the need to increase tax rates on goods with better tax collecting technologies and decrease rates on others.

Table 1. Quantitative Findings

	τ_K (%)	τ_W (%)	τ_C (%)	γ (%)	u (%)	v (%)	x (%)	$1-(u+v+x)$ (%)
Competitive Equilibrium*	18.5	2	19	3.73	33.28	14.55	7.5	44.67
Ramsey Allocation	10.78	-9.2	28.06	3.82	2.13	4.55	47.53	45.79
(*) See Appendix A.3								

Source: Authors' calculations.

Second, optimal allocation translates into a significant reallocation of time, not only between formal and informal work, but also between work and leisure. However, the huge reallocation of time is from work (total hours in both sectors) to nonmarket activities. Remarkably, the amount of time devoted to accumulate human capital along both BGPs is almost unchanged. In this sense, we observe that, in spite of the need for considerable tax rate changes, the growth rate only increases 0.09 percent, from 3.73 to 3.82 percent. A similar effect was cited early on Lucas (1990). This may appear puzzling a priori, because one could presume that the needed tax changes would foster both physical and human capital accumulation and then economic growth. However, the significant increase needed in the consumption tax would lead to greater consumption of untaxed leisure. As a result, the time devoted to accumulate human capital (and thus the growth rate along the BGP) would remain basically unchanged. It is important interpret leisure in a broad sense, as modeled, and include nonmarket activities (i.e., the production of nonmarket goods). Still, the large increase in those activities deserves further study.

We consider these results key to understanding, from a different perspective, some of the ideas behind radical fiscal reforms. For instance, Anton, Hernandez and Levy (2012) propose a provocative fiscal reform for Mexico to mitigate the harmful effects of informality on the labor market. Surprisingly, since our setting is not targeted a priori to match any important feature of the Mexican economy, our predictions are in line with these authors' proposal, not only qualitatively but also quantitatively. Their proposed reform would shift taxation to cover social insurance from labor to consumption, eliminating labor taxes and setting a uniform value added tax rate of 16 percent. Our results indicate that this proposal, under some circumstances, might indeed fall short. However, this is simply indicative and a more careful, in-depth study is necessary.

Marginal Tax Collecting Technological Changes

This section concludes with two exercises in which we vary, only marginally, the parameters of the tax collecting technologies at the time. Table 2 displays zoomed sections of Figures 7 and 8, in which e_W is kept at its calibrated value, 0.82, and $b = -0.13$, while e_K moves around its calibrated value, 0.69. As the technology to collect taxes on capital improves (i.e., e_K increases), the optimal capital tax rate increases, labor becomes even more subsidized, and consumption is taxed more heavily.

Table 2. Effects of Marginal Changes to Tax Collecting Technology (Scenario I)

$e_W = 0.82 \mid e_K = ...$	τ^K (%)	τ^W (%)	τ^C (%)
0.683	10.26	-8.19	20.63
0.686	10.53	-8.73	24.07
0.690	**10.78**	**-9.20**	**28.06**
0.692	10.93	-9.49	30.17
0.695	11.14	-9.92	33.32

Source: Authors' calculations.

Table 3 also displays zoomed sections of Figures 7 and 8, in which $e_K = 0.69$ and $b = -0.13$, while e_K moves marginally around its calibrated value, 0.82. As the technology to collect optimal taxes improves, the optimal capital tax rate decreases, labor becomes less subsidized and consumption is taxed less heavily.

Table 3. Effects of Marginal Changes to Tax Collecting Technology (Scenario II)

$e_K = 0.69 \mid e_W = ...$	τ^K (%)	τ^W (%)	τ^C (%)
0.790	11.18	-10.1	31.56
0.796	11.07	-9.82	30.64
0.806	10.98	-9.53	29.53
0.815	10.86	-9.34	28.77
0.820	**10.78**	**-9.20**	**28.06**
0.831	10.65	-8.93	26.88
0.839	10.54	-8.71	25.92

Source: Authors' calculations.

Around the calibrated levels for e_K and e_W, the optimal tax rates change as expected. The better the tax collecting technology, the relatively higher the corresponding tax. Moreover, these exercises show that fiscal reforms to improve efficiency in tax collection would have a significant impact on all optimal tax rates, in particular consumption taxes. In this case, proposals for fiscal reforms to make radical changes would indeed be exacerbated.

Both exercises shed light on two remarkable facts for this set of parameters. First, capital and consumption taxes are complementary; that is, changes in tax collecting technologies will make both rates move in the same direction. Second, efficiency dictates that labor taxes should be significantly subsidized, reflecting the fact that workers in the formal sector are more productive, and so the government should optimally allocate more resources to that sector.

7 Final Remarks

This paper has made progress in characterizing competitive equilibrium and Ramsey allocations in the context of a small open economy with the following conditions: the interest rate is endogenously determined; some workers can be hired in the informal market; and tax collecting technologies are imperfect and heterogeneous for different taxes. We have addressed two questions in this setting. The first is Ramsey's (1927) normative question: What choice of tax rates will maximize consumer utility, consistent with given government consumption and with market determination of quantities and prices? The second is positive and quantitative: How much difference does it make?

Our quantitative exercises show that, from a baseline economy, the inclusion of an informal sector reduces the growth rate over the BGP. Increasing labor taxes produces a reduction in the time devoted to both work in the formal sector and to human capital accumulation. An increase in capital taxes has the same effect, albeit less pronounced.

Optimal taxes stemming from the Ramsey allocation suggests that capital and labor taxes increase according to the level of efficiency of their corresponding tax collecting technologies. On the other hand, in response to changes in technology efficiency, consumption tax rates move in the same direction as capital tax rates.

In sum, the policy recommendations stemming from our quantitative results for the parameters calibrated to the Chilean economy suggest that capital–which should not be taxed according to standard neoclassical growth models–has to be taxed, albeit at a considerably decreased rate compared to the observed one (that is, 10.78 percent compared to 18.5 percent). Labor should be subsidized (to stimulate human capital accumulation), while consumption taxes should be increased by 50 percent (from 19 to 28 percent). The resulting growth rate increases only slightly along the BGP, despite quite significant changes in time devoted to both formal and informal labor, as well as to leisure activities.

References

Anton, A., F. Hernández and S. Levy. 2012. "The End of Informality in Mexico? Fiscal Reform for Universal Social Insurance." IDB Monograph Series No. IDB-MG-146. Washington, DC: Inter-American Development Bank.

Arrau, P. 1990. "Intertemporal Substitution in a Monetary Framework." World Bank Working Paper Series 549. Washington, DC: World Bank.

Atkinson, A. B., and J. E. Stiglitz. 1972. "The Structure of Indirect Taxation and Economic Efficiency." *Journal of Public Economics* 1: 97–119.

Chamley, Christophe. 1986. "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives." *Econometrica* 54 (May): 607–22.

Chari, V. V., L. J. Christiano, and E. J. Kehoe. 1991. "Optimal Fiscal and Monetary Policy: Some Recent Results." *Journal of Money, Credit, and Banking* 23: 519–39.

Correia, I. H. 1996. "Dynamic Optimal Taxation in Small Open Economies." *Journal of Economic Dynamics and Control* 20(4): 691–708.

Jones, L. E., R. E. Manuelli, and P. E. Rossi. 1993. "Optimal Taxation in Models of Endogenous Growth." *Journal of Political Economy* 101: 485–517.

Jorrat, M. 2012. "Gastos tributarios y evasión tributaria en Chile: evaluación y propuestas." Santiago, Chile: Centro de estudios de Chile.

King, R. G. and S. Rebelo. 1990. "Public Policy and Endogenous Growth: Developing Neoclassical Implications." *Journal of Political Economy* 98: 126–50.

Laitner, J. 1995. "Quantitative Evaluations of Efficient Tax Policies for Lucas' Supply Side Models." *Oxford Economic Papers* 47: 471–92.

Lucas, R. E. 1990. "Supply-side Economics: An Analytical Review." *Oxford Economic Papers* 42: 293–316.

Lucas Jr., R. E., and N. L. Stokey. 1983. "Optimal Fiscal and Monetary Policy in an Economy without Capital." *Journal of Monetary Economics* 12: 55–93.

Perez Toledo. 2003. "Stock de capital de la economía chilena y su distribución sectorial." Central Bank of Chile Working Papers 233. Santiago, Chile: Central Bank of Chile.

Stokey, N. L. and S. Rebelo. 1995. "Growth Effects of Flat-rate Taxes." *Journal of Political Economy* 103: 519–50.

Schmitt-Grohe, S. and M. Uribe. 2003. "Closing Small Open Economy Models." *Journal of International Economics* 61(1): 163–85.

Appendix

In this Appendix, we characterize first a competitive equilibrium (Subsection A.1) and then the Ramsey problem (Subsection A.2).

A.1 Competitive Equilibrium

The representative household solves

$$\max_{\{C_t, u_t, v_t, K_{t+1}, H_{t+1}, B^p_{t+1}\}} \sum_{t=0}^{\infty} \rho^t \, \frac{\left(C_t \ (x_t)^\theta\right)^{1-\sigma}}{1-\sigma}, \tag{7}$$

subject to

$$\begin{aligned} [\lambda_t] \quad : \qquad & (1+\tau_C)C_t + (K_{t+1} - K_t) + B^p_{t+1} \\ = \quad & (1-\tau_W)\, w^F_t \ u_t \ H_t + w^I_t \ v_t \ H_t \\ & + (1-\tau_K) \ \ (r_t - \delta_K) \ K_t + (1+R_t)\, B^p_t \end{aligned} \tag{8}$$

$$[\mu_t]: \qquad H_{t+1} = A^H \ (1 - u_t - v_t - x_t) \ H_t + (1-\delta_H) \ H_t \tag{9}$$

$$\lim_{T\to\infty} \prod_{j=0}^{T} \frac{B^p_T}{(1+R_j)} \geq 0,$$

where (K_0, H_0, B_0) are given, and the last condition restricts $\left\{B^g_{t+1}\right\}_{t=0}^{\infty}$ to rule out Ponzi schemes. The brackets include associated Lagrange multipliers. The conditions characterizing a solution are

$$(C_t): \rho^t \ \left[C_t \ x_t^\theta\right]^{-\sigma} x_t^\theta = (1+\tau^c_t)\lambda_t \tag{10}$$

$$(x_t): \rho^t \ \left[C_t x_t^\theta\right]^{-\sigma} \ C_t \ \theta \ x_t^{\theta-1} = \mu_t \ A^H \ H_t$$

$$(K_{t+1}): \lambda_t = \lambda_{t+1}\left[1 + (r_{t+1} - \delta_K)\left(1-\tau^K\right)\right] \tag{11}$$

$$(u_t): \mu_t \ A^H \ H_t = \lambda_t \ (1-\tau^w) \ H_t \ w^F_t \tag{12}$$

$$(v_t): \mu_t \ A^H \ H_t = \lambda_t \ H_t \ w^I_t \tag{13}$$

$$\begin{aligned} (H_{t+1}) \quad : \quad & \mu_t = \mu_{t+1} \ \left[A^H \ \ (1 - u_{t+1} - v_{t+1} - x_{t+1}) + (1-\delta_H)\right] \\ & + \lambda_{t+1}\left[(1-\tau^w_{t+1}) \ w^F_{t+1} u_{t+1} + w^I_{t+1} \ v_{t+1}\right] \end{aligned} \tag{14}$$

$$\left(B^p_{t+1}\right): \lambda_t = \lambda_{t+1} \ \ (1 + R_{t+1}) \tag{15}$$

$$\left(TCB_{t+1}^{p}\right) : \lim_{T\to\infty} \lambda_T B_T^p = \lim \prod_{\substack{j=0 \\ T\to\infty}}^{T} \frac{B_T^p}{(1+R_j)} = 0 \tag{16}$$

$$(TCK_{t+1}) : \lim_{T\to\infty} \lambda_T K_T = \lim_{T\to\infty} \prod_{j=0}^{T} \frac{K_T}{(1+R_j)} = 0 \tag{17}$$

$$\begin{aligned} (\lambda_t) \quad &: \quad (1+\tau_t^c)C_t + (K_{t+1} - K_t) + B_{t+1}^p \\ &= \quad (1-\tau_t^w)w_t^F u_t\, H_t + w_t^I v_t\, H_t + \left(1-\tau_t^k\right)(r_t - \delta_K)\, K_t + (1+R_t)\, B_t^p \end{aligned} \tag{18}$$

$$(\mu_t) : H_{t+1} = A^H\ (1-u_t-v_t-x_t)\ H_t + (1-\delta_H)\ H_t, \tag{19}$$

where

$$\begin{aligned} w_t^F &= F_2(K_t, u_t H_t, v_t H_t) \\ &= A\,(K_t)^{\alpha}\,(1-\alpha)\left(\beta\ (u_t H_t)^{\frac{\eta-1}{\eta}} + (1-\beta)\ (v_t H_t)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta(1-\alpha)}{\eta-1}-1} \beta\,(u_t H_t)^{-\frac{1}{\gamma}} \end{aligned}$$

$$\begin{aligned} w_t^I &= F_2(K_t, u_t H_t, v_t H_t) \\ &= A\,(K_t)^{\alpha}\,(1-\alpha)\left(\beta\ (u_t H_t)^{\frac{\eta-1}{\eta}} + (1-\beta)\ (v_t H_t)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta(1-\alpha)}{\eta-1}-1} (1-\beta)\,(v_t H_t)^{-\frac{1}{\gamma}}\,. \end{aligned}$$

The result from (12) and (13) is

$$(1-\beta)\,(v_t)^{-\frac{1}{\eta}} = (1-\tau^w)\,\beta\,(u_t)^{-\frac{1}{\eta}}\,.$$

The budget constraints of the agent and the government are coupled together to obtain

$$\begin{aligned} &(1+\tau^c\,(1-e_c))\frac{C_t}{H_t} + \left(\frac{K_{t+1}}{K_t} - 1\right)\frac{K_t}{H_t} + \frac{B_{t+1}}{B_t}\frac{B_t}{H_t} + g\frac{Y_t}{H_t} \\ = \quad &(1-\tau^w\,(1-e_c))\, w_t^F\ u_t + w_t^I v_t + \left(1-\tau^k\,(1-e_c)\right)(r_t-\delta_K)\,\frac{K_t}{H_t} + (1+R_t)\,\frac{B_t}{H_t}. \end{aligned} \tag{20}$$

Conditions (10) - (20) characterize a competitive equilibrium.

Balanced Growth Analysis: Competitive Equilibrium

The following equations characterize the balanced growth path (BGP) for the economy modeled above:

$$\rho\left(\gamma\right)^{-\sigma}=\left(1+R\left(\frac{b^{p}+b^{g}}{k}\right)\right)^{-1}$$

$$\left(\frac{\mu}{\lambda}\right)\ x=c\,\frac{\left(1+\tau^{c}\right)}{A^{H}}\,\theta$$

$$1=\left(1+R\left(\frac{b^{p}+b^{g}}{k}\right)\right)^{-1}\left[1+\left(r-\delta_{K}\right)\left(1-\tau^{K}\right)\right]$$

$$\left(\frac{\mu}{\lambda}\right)=\frac{A}{A^{H}}\left(1-\tau^{w}\right)w^{F}$$

$$\left(1-\beta\right)^{\eta}\ u=\left(1-\tau^{w}\right)^{\eta}\beta^{\eta}\,v$$

$$\begin{aligned}1\ &=\ \gamma^{*}\ \left(1+R\left(\frac{b^{p}+b^{g}}{k}\right)\right)^{-1}\\&\quad+\left(1+R\left(\frac{b^{p}+b^{g}}{k}\right)\right)^{-1}\frac{\lambda}{\mu}\left(\left(1-\tau^{w}\right)w^{F}\ u+w^{I}\ v\right)\end{aligned}$$

$$\gamma^{*}=A^{H}\ \left(1-u-v-x\right)+\left(1-\delta_{H}\right)$$

$$r=\alpha\,A\,k^{\alpha}\left(\beta\ \left(u\right)^{\frac{\eta-1}{\eta}}+\left(1-\beta\right)\ \left(v\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta(1-\alpha)}{\eta-1}}$$

$$\begin{aligned}&\left(1+\tau^{c}\left(1-e_{c}\right)\right)c+\left(\gamma-1\right)\ k+g\,A\,k^{\alpha}\left(\beta\ \left(u\right)^{\frac{\eta-1}{\eta}}+\left(1-\beta\right)\ \left(v\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta(1-\alpha)}{\eta-1}}\\=\ &\left(1-\tau^{w}\left(1-e_{w}\right)\right)w^{F}\ u+w^{I}\ v+\left(1-\tau^{k}\left(1-e_{k}\right)\right)\left(r-\delta_{K}\right)k\\&+\left(1+R\left(\frac{b^{p}+b^{g}}{k}\right)-\gamma\right)\left(b^{p}+b^{g}\right)\end{aligned}$$

$$\begin{aligned}&g\,A\,k^{\alpha}\left(\beta\ \left(u\right)^{\frac{\eta-1}{\eta}}+\left(1-\beta\right)\ \left(v\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta(1-\alpha)}{\eta-1}}\\=\ &e_{c}\,\tau^{C}\ c+e_{w}\,\tau^{W}\ w^{F}\ u+e_{k}\,\tau^{K}\ \left(r-\delta_{K}\right)\ k\\&+\left(1+R\left(\frac{b^{p}+b^{g}}{k}\right)-\gamma\right)\ b^{g},\end{aligned}$$

where

$$w^{F}\ =\ Ak^{\alpha}\left(1-\alpha\right)\left(\beta\ \left(u\right)^{\frac{\eta-1}{\eta}}+\left(1-\beta\right)\ \left(v\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta(1-\alpha)}{\eta-1}-1}\beta\left(u\right)^{-\frac{1}{\eta}}$$

$$w^{I}\ =\ Ak^{\alpha}\left(1-\alpha\right)\left(\beta\ \left(u\right)^{\frac{\eta-1}{\eta}}+\left(1-\beta\right)\ \left(v\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta(1-\alpha)}{\eta-1}-1}\left(1-\beta\right)\left(v\right)^{-\frac{1}{\eta}}.$$

The unknowns of this system are $\left(\gamma, \frac{\mu}{\lambda}, r, k, x, u, v, c, b^p, b^g\right)$. Lower case letters denote the corresponding variables in terms of H.

A.2 The Ramsey Problem

To apply the *primal approach*, multiply the budget constraint (18) by λ_t, and add them up to date T to get

$$\begin{aligned}
& \sum_{t=0}^{T} \lambda_t (1+\tau_t^c) C_t + \sum_{t=1}^{T-1} \left(\lambda_{t-1} - \lambda_t \left[1 + \left(1-\tau_t^k\right)(r_t - \delta_K)\right]\right) K_t + \lambda_T\, K_{T+1} \\
& + \sum_{t=1}^{T-1} \left(\lambda_{t-1} - \lambda_t \left(1+R_t\right)\right)\ B_t^p + \lambda_T B_{T+1}^p \\
= \; & \sum_{t=0}^{T} \lambda_t\ \left((1-\tau_t^w) w_t^F u_t + w_t^I v_t\right)\ H_t + \lambda_0 \left[1 + \left(1-\tau_0^k\right)(r_0 - \delta_K)\right] K_0 + \lambda_0 \left(1+R_0\right) B_0^p.
\end{aligned}$$

Notice that

$$\begin{aligned}
& \lambda_{t+1}\ \left((1-\tau_{t+1}^w) w_{t+1}^F u_{t+1} + w_{t+1}^I v_{t+1}\right)\ H_{t+1} \qquad (21)\\
= \; & \mu_{t+1}\ \left[A^H\ (1 - u_{t+1} - v_{t+1} - x_{t+1}) + (1-\delta_H)\right] H_{t+1} - \mu_t\, H_{t+1} \\
= \; & \mu_{t+1}\, H_{t+2} - \mu_t\, H_{t+1}.
\end{aligned}$$

Hence, using the conditions characterizing a competitive equilibrium, and taking the limit as $T \to \infty$, the last expression reduces to

$$\begin{aligned}
& \sum_{t=0}^{\infty} \rho^t\, U_C(C_t, x_t)\, C_t \qquad (22)\\
= \; & \frac{U_C(C_0, x_0)}{(1+\tau_0^c)}\ \left[F_3\left(K_0, u_0 H_0, v_0 H_0\right)\ \left(u_0+v_0\right) H_o\right. \\
& \left. + \left[1 + \left(1-\tau_0^k\right)(r_0 - \delta_K)\right] K_0 + \left(1+R_0\right)\ B_0^p\right] \qquad (23)\\
= \; & W_0,
\end{aligned}$$

where $U_C(C_t, x_t) = \left(C_t\, x_t^\theta\right)^{-\sigma} x_t^\theta$ for all t.

Also, equation (21) reduces to

$$U_C(C_t, x_t) = \rho\, U_C(C_{t+1}, x_{t+1})\, w_{t+1}^I\ \left[A^H (1 - x_{t+1}) + (1-\delta_H)\right]$$

and

$$\tau_t^w = \left(\frac{w_t^F - w_t^I}{w_t^F} \right)$$

$$\tau_t^K = \left(\frac{r_t - \delta_K - R_t}{r_t - \delta_K} \right).$$

Without loss of generality, we normalize $e_c = 1$ and denote

$$\begin{aligned} taow_t &\equiv \left(\frac{F_2(t) - F_3(t)}{F_2(t)} \right) \\ taor_t &\equiv \left(\frac{F_1(t) - \delta_K - R_t}{F_1(t) - \delta_K} \right). \end{aligned}$$

Let ϕ be the Lagrange multiplier corresponding to the incentive constraint (22) and define

$$\begin{aligned} V(C_t, x_t; \phi) &\equiv U(C_t, x_t) + \phi \, U_C(C_t, x_t) \, C_t \\ &= (1 + (1 - \sigma) \, \phi) \, \frac{\left(C_t \, (x_t)^\theta \right)^{1-\sigma}}{1 - \sigma}. \end{aligned}$$

The Ramsey problem for this economy is

$$\max_{(C_t, u_t, v_t K_{t+1}, H_{t+1}, B_{t+1})} \sum_{t=0}^{\infty} \rho^t \, V(C_t, x_t; \phi) \; - \; \phi \, W_0,$$

subject to

$$U_C(C_t, x_t) = \rho \, U_C(C_{t+1}, x_{t+1}) \, w_{t+1}^I \, \left[A^H (1 - x_{t+1}) + (1 - \delta_H) \right]$$

$$H_{t+1} = A^H \, (1 - u_t - v_t - x_t) \, H_t + (1 - \delta_H) H_t$$

$$\begin{aligned} & C_t + (K_{t+1} - K_t) + B_{t+1} + g \, Y_t \\ = \; & [1 - taow_t \, (1 - e_w)] \, F_2(t) \, u_t \, H_t + F_3(t) \, v_t \, H_t + [1 - taor_t \, (1 - e_k)] \, (F_1(t) - \delta_K) \, K_t \\ & + (1 + R_t) \, B_t. \end{aligned}$$

We denote the corresponding (date t) Lagrange multipliers by $\rho^t \chi_t^1$, $\rho^t \chi_t^2$, and $\rho^t \chi_t^3$, respectively. First order conditions are given by

$$C_t : \chi_t^3 = V_C(C_t, x_t; \phi) + U_{CC}(C_t, x_t) \left[\chi_t^1 - \chi_{t-1}^1 \, F_3(t) \, \left[A^H (1 - x_t) + (1 - \delta_H) \right] \right]$$

$$
\begin{aligned}
x_t \quad &: \quad \chi_t^2 A^H H_t \\
&= \quad V_x(C_t, x_t; \phi) + \chi_t^1 \, U_{C,x}(C_t, x_t) \\
&\quad\; -\chi_{t-1}^1 \left[U_{C,x}(C_t, x_t) \ \left(A^H \ (1 - x_t) + (1 - \delta_H) \right) - A^H \, U_C(C_t, x_t) \right]
\end{aligned}
$$

$$
\begin{aligned}
u_t \quad : \quad & \chi_t^2 \, A^H \, H_t + \chi_{t-1}^1 \, \frac{\partial F_3(t)}{\partial u_t} \, U_C(C_t, x_t) \ \left[A^H (1 - x_t) + (1 - \delta_H) \right] = \\
& + \chi_t^3 \Bigg[(1 - taow_t \ (1 - e_w)) \ \left(\frac{\partial F_2(t)}{\partial u_t} \, u_t \, H_t + F_2(t) \, H_t \right) \\
& - \frac{\partial taow_t}{\partial u_t} \ (1 - e_w) \ F_2(t) \, u_t \, H_t \\
& \frac{\partial F_3(t)}{\partial u_t} \, v_t \, H_t - \frac{\partial taor_t}{\partial u_t} \ (1 - e_c) \ (F_1(t) - \delta_K) \ K_t \\
& + (1 - taor_t \ (1 - e_c)) \ \frac{\partial F_1(t)}{\partial u_t} \, K_t - g \frac{\partial Y(t)}{\partial u_t} \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
v_t \quad : \quad & \chi_t^2 \, A^H \, H_t + \chi_{t-1}^1 \, \frac{\partial F_3(t)}{\partial v_t} \, U_C(C_t, x_t) \ \left[A^H (1 - x_t) + (1 - \delta_H) \right] = \\
& \chi_t^3 \ \Bigg[[1 - taow_t \ (1 - e_w)] \ \frac{\partial F_2(t)}{\partial u_t} \, u_t \, H_t \\
& - \frac{\partial taow_t}{\partial v_t} \ (1 - e_w) \ F_2(t) \, u_t \, H_t \\
& + \frac{\partial F_3(t)}{\partial v_t} \, v_t \, H_t + F_3(t) \, H_t - \frac{\partial taor_t}{\partial v_t} \ (1 - e_c) \ (F_1(t) - \delta_K) \ K_t \\
& + [1 - taor_t \ (1 - e_c)] \ \frac{\partial F_1(t)}{\partial v_t} \, K_t - g \frac{\partial Y(t)}{\partial v_t} \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
K_{t+1} \quad : \quad & \chi_{t+1}^3 + \chi_t^1 \, \frac{\partial F_3(t+1)}{\partial K_{t+1}} \, \rho \, U_C(C_{t+1}, x_{t+1}) \ \left(A^H \ (1 - x_{t+1}) + (1 - \delta_H) \right) = \\
& \rho \, \chi_{t+1}^3 \Bigg[- \frac{\partial taow_{t+1}}{\partial K_{t+1}} \ (1 - e_w) \ F_2(t+1) \, u_{t+1} \, H_{t+1} \\
& + [1 - taow_{t+1} \ (1 - e_w)] \ \frac{\partial F_2(t+1)}{\partial K_{t+1}} \, u_{t+1} \, H_{t+1} \\
& + \frac{\partial F_3(t+1)}{\partial K_{t+1}} \, v_{t+1} \, H_{t+1} - \frac{\partial taor_{t+1}}{\partial K_{t+1}} \ (1 - e_c) \ (F_1(t+1) - \delta_K) \ K_{t+1} \\
& + (1 - taor_t \ (1 - e_c)) \ \left(\frac{\partial F_1(t+1)}{\partial K_{t+1}} \, K_{t+1} + (F_1(t+1) - \delta_K) \right) + 1 - g \, \frac{\partial Y(t+1)}{\partial K_{t+1}} \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
H_{t+1} \quad : \quad & \chi_t^2 + \chi_t^1 \, \frac{\partial F_3(t+1)}{\partial H_{t+1}} \, \rho \, U_C(C_{t+1}, x_{t+1}) \, \left(A^H \ (1 - x_{t+1}) + (1 - \delta_H)\right) = \\
& \rho \, \chi_{t+1}^2 \left(A^H \ (1 - u_{t+1} - v_{t+1} - x_{t+1}) + (1 - \delta_H)\right) \\
& + \rho \, \chi_{t+1}^3 \left[- \frac{\partial taow_{t+1}}{\partial H_{t+1}} \ (1 - e_w) \ F_2(t+1) \, u_{t+1} \, H_{t+1} \right. \\
& + (1 - taow_{t+1} \ (1 - e_w)) \ \left(\frac{\partial F_2(t+1)}{\partial H_{t+1}} \, u_{t+1} \, H_{t+1} + F_2(t+1) \, u_{t+1} \right) \\
& + v_{t+1} H_{t+1} \frac{\partial F_3(t+1)}{\partial H_{t+1}} + v_{t+1} F_3(t+1) - \frac{\partial taor_{t+1}}{\partial H_{t+1}} \ (1 - e_c) \ \left(F_1(t+1) - \delta_K\right) \ K_{t+1} \\
& \left. + (1 - taor_{t+1} \ (1 - e_c)) \ \frac{\partial F_1(t+1)}{\partial H_{t+1}} \, K_{t+1} - g \, \frac{\partial Y(t+1)}{\partial H_{t+1}} \right]
\end{aligned}
$$

$$
B_{t+1} : \chi_t^3 = \rho \, \chi_{t+1}^3 \left(1 + R\left(\frac{B_{t+1}}{K_{t+1}}\right)\right)
$$

$$
U_C(C_t, x_t) = \rho \, U_C(C_{t+1}, x_{t+1}) \, w_{t+1}^I \ \left[A^H (1 - x_{t+1}) + (1 - \delta_H)\right]
$$

$$
H_{t+1} = A^H \, (1 - u_t - v_t - x_t) \, H_t + (1 - \delta_H) H_t
$$

$$
\begin{aligned}
& C_t + (K_{t+1} - K_t) + B_{t+1} + g \, Y_t \\
= \quad & [1 - taow_t \ (1 - e_w)] \ F_2(t) \, u_t \, H_t + F_3(t) \, v_t \, H_t + [1 - taor_t \ (1 - e_c)] \ \left(F_1(t) - \delta_K\right) \ K_t \\
& + (1 + R_t) \ B_t.
\end{aligned}
$$

Balanced Growth Analysis: Ramsey Allocation

Notice that

$$
U(C_t, x_t) = \frac{\left(C_t \ (x_t)^\theta\right)^{1-\sigma}}{1 - \sigma}
$$

$$
U_C(C_t, x_t) = (C_t)^{-\sigma} \, (x_t)^{\theta(1-\sigma)}
$$

$$
\begin{aligned}
U_x(C_t, x_t) \quad &= \quad \theta \ (C_t)^{1-\sigma} \, (x_t)^{\theta(1-\sigma)-1} \\
&= \quad \theta \, \frac{C_t}{x_t} \, U_C(C_t, x_t)
\end{aligned}
$$

$$
U_{CC}(C_t, x_t) = -\sigma \, (C_t)^{-\sigma-1} \, (x_t)^{\theta(1-\sigma)} = -\sigma \, (C_t)^{-1} \, U_C(C_t, x_t)
$$

$$
\begin{aligned}
U_{C,x}(C_t, x_t) &= \theta\,(1-\sigma)\,(C_t)^{-\sigma}\,(x_t)^{\theta(1-\sigma)-1} \\
&= \theta\,(1-\sigma)\,(x_t)^{-1}\,U_C(C_t, x_t),
\end{aligned}
$$

and

$$
\begin{gathered}
V(C_t, x_t; \phi) = (1 + (1-\sigma)\,\phi)\;\frac{\left(C_t\;(x_t)^{\theta}\right)^{1-\sigma}}{1-\sigma} \\
V_C(C_t, x_t; \phi) = (1 + (1-\sigma)\,\phi)\;U_C(C_t, x_t) \\
V_x(C_t, x_t; \phi) = (1 + (1-\sigma)\,\phi)\;U_x(C_t, x_t) = \theta\,\frac{C_t}{x_t}\,U_C(C_t, x_t) \\
V_{C,x}(C_t, x_t; \phi) = (1 + (1-\sigma)\,\phi)\;\theta\,(1-\sigma)\,(x_t)^{-1}\,U_C(C_t, x_t).
\end{gathered}
$$

Based on a detailed analysis, the following conditions characterize a BGP of the Ramsey allocation:

$$
z^* \;=\; (1 + (1-\sigma)\;\phi) - \sigma\,\frac{1}{c^*}\,p^*\;\left(1 - \frac{1}{\gamma^*}\,F_3^*\;\left[A^H(1-x^*) + (1-\delta_H)\right]\right)
$$

$$
\begin{aligned}
m^* A^H \;=\;& \theta\,c^*\;(x^*)^{-1} + p^*\,\theta\,(1-\sigma)\,(x^*)^{-1} \\
& - \frac{1}{\gamma^*}p^*\left[\theta\,(1-\sigma)\,(x^*)^{-1}\left(A^H\;(1-x^*) + (1-\delta_H)\right) - A^H\right]
\end{aligned}
$$

$$
\begin{aligned}
& m^*\,A^H + \frac{1}{\gamma^*}p^*\;(F_{32}\,H)^*\;\left[A^H(1-x^*) + (1-\delta_H)\right] \\
=\;& z^*\Bigg[(1 - taow^*\;(1-e_w))\;\left((F_{22}H)^*\;u^* + F_2^*\right) - \left(\frac{\partial taow}{\partial u}\right)^*\;(1-e_w)\;F_2^*\;u^* \\
& \quad * (F_{32}\,H)^*\;v^* - \left(\tfrac{\partial taor}{\partial u}\right)^*\;(1-e_c)\;(F_1^* - \delta_K)\;k^* \\
& \quad + (1 - taor^*\;(1-e_c))\;(F_{12}\,H)^*\;k^* - g\,F_2^* \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
& m^* \; A^H \; + \frac{1}{\gamma^*} p^* \; (F_{33} \; H_t)^* \; \left[A^H (1 - x^*) + (1 - \delta_H)\right] \\
= \; & z^* \; \Bigg[(1 - taow^* \; (1 - e_w)) \; (F_{22} \; H)^* \; u^* \; - \left(\frac{\partial taow}{\partial v}\right)^* \; (1 - e_w) \; F_2^* \; u^* \\
& + (F_{33} \; H_t)^* \; v^* + F_3^* - \left(\tfrac{\partial taor}{\partial v}\right)^* \; (1 - e_c) \; (F_1^* - \delta_K) \; k^* \\
& \quad + (1 - taor^* \; (1 - e_c)) \; (F_{13} \; H)^* \; k^* - g \; F_3^* \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
& z^* + \frac{1}{\gamma^*} p^* \; (F_{31} \; H)^* \; \rho \; \left(A^H \; (1 - x^*) + (1 - \delta_H)\right) \\
= \; & \rho \, z^* \left[(1 - taow^* \; (1 - e_w)) \; (F_{21} \; H_{t+1})^* \; u^* \right. \\
& + (F_{31} \; H)^* \; v^* - \left(\frac{\partial taor}{\partial K} K\right)^* \; (1 - e_c) \; (F_1^* - \delta_K) \\
& \left. + (1 - taor^* \; (1 - e_c)) \; ((F_{11} \; K)^* + (F_1^* - \delta_K)) + 1 - g \; F_1^* \right]
\end{aligned}
$$

$$
\begin{aligned}
& (\gamma^*)^\sigma \; m^* + \rho \, \frac{1}{\gamma^*} \, p^* \; ((F_{32} \; H)^* \; u^* + (F_{33} \; H)^* \; v^*) \; \left(A^H \; (1 - x^*) + (1 - \delta_H)\right) \\
= \; & \rho \, m^* \, \gamma^* + \rho \, z^* \left[(1 - taow^* \; (1 - e_w)) \; (((F_{22} \; H)^* \; u^* + (F_{23} \; H)^* \; v^* + F_2^*) \; u^*) \right. \\
& + ((F_{32} \; H)^* \; u^* + (F_{33} \; H)^* \; v^* + F_3^*) \; v^* - \left(\frac{\partial taor}{\partial H} \; H\right)^* \; (1 - e_c) \; (F_1^* - \delta_K) \, k^* \\
& \left. + (1 - taor^* \; (1 - e_c)) \; ((F_{12} \; H)^* \; u^* + (F_{13} \; H)^* \; v^*) \; k^* - g \; (F_2^* \, u^* + F_3^* \, v^*) \right]
\end{aligned}
$$

$$
(\gamma^*)^\sigma \;\; = \;\; \rho \; \left(1 + R\left(\frac{b^*}{k^*}\right)\right)
$$

$$
(\gamma^*)^\sigma \;\; = \;\; \rho \, F_3^* \; \left[A^H (1 - x^*) + (1 - \delta_H)\right]
$$

$$
\gamma^* \;\; = \;\; A^H \, (1 - u^* - v^* - x^*) + (1 - \delta_H)
$$

$$\begin{aligned}
& c^* + (\gamma^* - 1)\, k^* + g\, y^* \\
= \;& [1 - taow^* \; (1 - e_w)] \; F_2^* \, u^* + F_3^* \, v^* + [1 - taor^* \; (1 - e_c)] \; (F_1^* - \delta_K) \; k^* \\
& + \left(1 + R\left(\frac{b^*}{k^*}\right) - \gamma^*\right) \; b^*,
\end{aligned}$$

where

$$\begin{aligned}
F_1^* &= (1 - \alpha - \beta)\, A\, (k^*)^{-\alpha-\beta}\, (u^*)^{\alpha}\, (v^*)^{\beta} \\
F_2^* &= \alpha A\, (k^*)^{1-\alpha-\beta}\, (u^*)^{\alpha-1}\, (v^*)^{\beta} \\
F_3^* &= \beta A\, (k^*)^{1-\alpha-\beta}\, (u^*)^{\alpha}\, (v^*)^{\beta-1} \\
(F_{11}\, K)^* &= -\,(1 - \alpha - \beta)\,(\alpha + \beta) A\, (k^*)^{-(\alpha+\beta)}\, (u^*)^{\alpha}\, (v^*)^{\beta} \\
(F_{12}\, H)^* &= \alpha\,(1 - \alpha - \beta)\, A\, (k^*)^{-\alpha-\beta}\, (u^*)^{\alpha-1}\, (v^*)^{\beta} \\
(F_{13}\, H)^* &= \beta\,(1 - \alpha - \beta)\, A\, (k^*)^{-\alpha-\beta}\, (u^*)^{\alpha}\, (v^*)^{\beta-1} \\
(F_{22}\, H)^* &= -\alpha\,(1 - \alpha)\, A\, (k^*)^{1-\alpha-\beta}\, (u^*)^{\alpha-2}\, (v^*)^{\beta} \\
(F_{33}\, H)^* &= -\beta\,(1 - \beta)\, A\, (k^*)^{1-\alpha-\beta}\, (u^*)^{\alpha}\, (v^*)^{\beta-2} \\
(F_{32}\, H)^* &= \alpha\beta A\, (k^*)^{1-\alpha-\beta}\, (u^*)^{\alpha-1}\, (v^*)^{\beta-1}
\end{aligned}$$

$$\begin{aligned}
taow^* &\equiv \left(\frac{\alpha\,(u^*)^{-1} - \beta\,(v^*)^{-1}}{\alpha\,(u^*)^{-1}}\right) = 1 - \frac{\beta}{\alpha}\frac{u^*}{v^*} \\
\left(\frac{\partial taow}{\partial u}\right)^* &= -\frac{\beta}{\alpha}\frac{1}{v^*} \\
\left(\frac{\partial taow}{\partial v}\right)^* &= \frac{\beta}{\alpha}\frac{u^*}{(v^*)^2} \\
\frac{\partial taow_t}{\partial K_t} &= \frac{\partial taow_t}{\partial H_t} = 0
\end{aligned}$$

$$
\begin{aligned}
taor^* &\equiv \left(\frac{(1-\alpha-\beta)\, A\, (k^*)^{-\alpha-\beta}\, (u^*)^{\alpha}\, (v^*)^{\beta} - \delta_K - R^*}{(1-\alpha-\beta)\, A\, (k^*)^{-\alpha-\beta}\, (u^*)^{\alpha}\, (v^*)^{\beta} - \delta_K} \right) \\
\left(\frac{\partial taor}{\partial u} \right)^* &= \frac{R^*\, \alpha\, (1-\alpha-\beta)\, A\, (k^*)^{-\alpha-\beta}\, (u^*)^{\alpha-1}\, (v^*)^{\beta}}{\left[(1-\alpha-\beta)\, A\, (k^*)^{-\alpha-\beta}\, (u^*)^{\alpha}\, (v^*)^{\beta} - \delta_K \right]^2} \\
\left(\frac{\partial taor}{\partial v} \right)^* &= \frac{R^*\, \beta\, (1-\alpha-\beta)\, A\, (k^*)^{-\alpha-\beta}\, (u^*)^{\alpha}\, (v^*)^{\beta-1}}{\left[(1-\alpha-\beta)\, A\, (k^*)^{-\alpha-\beta}\, (u^*)^{\alpha}\, (v^*)^{\beta} - \delta_K \right]^2} \\
\left(\frac{\partial taor}{\partial K}\, K \right)^* &= -\frac{R^*\ (1-\alpha-\beta)\, (\alpha+\beta) A\, (k^*)^{-\alpha-\beta}\, (u^*)^{\alpha}\, (v^*)^{\beta}}{\left[(1-\alpha-\beta)\, A\, (k^*)^{-\alpha-\beta}\, (u^*)^{\alpha}\, (v^*)^{\beta} - \delta_K \right]^2} \\
\left(\frac{\partial taor}{\partial H}\, H \right)^* &= \frac{R^* \begin{pmatrix} \alpha\, (1-\alpha-\beta)\, A\, (k^*)^{-\alpha-\beta}\, (u)^{\alpha}\, (v)^{\beta} \\ +\beta\, (1-\alpha-\beta)\, A\, (k^*)^{-\alpha-\beta}\, (u^*)^{\alpha}\, (v^*)^{\beta} \end{pmatrix}}{\left[(1-\alpha-\beta)\, A\, (k^*)^{-\alpha-\beta}\, (u^*)^{\alpha}\, (v^*)^{\beta} - \delta_K \right]^2},
\end{aligned}
$$

and

$$
\begin{aligned}
R^* &= R\left(\frac{b^*}{k^*} \right) \\
p^* &= \frac{\chi^1}{H} \\
m^* &= \frac{\chi^2}{U_C} \\
z^* &= \frac{\chi^3}{U_C}.
\end{aligned}
$$

We have a system of 10 equations and 10 unknowns: $(p^*, m^*, z^*, c^*, \gamma^*, k^*, u^*, v^*, x^*, b^*)$. Alternatively, we calibrate b^* and include ϕ^* as an unknown (i.e., the Lagrange multiplier corresponding to the implementation constraint).

A.3 Calibration

Before presenting the parameters values, we describe the ones used. The following table summarizes the calibration done for Chile:

The value of A was just a normalization. We chose A^H to match an annual growth rate of 3 percent. We chose the value of $1-\alpha$ to match the share of labor income in GNP according to national accounts data. The value of β is standard in the literature. For g, we used the average of the government spending to GDP ratio from 1960 to 2000. As for δ, we calculated it using the gross and net capital stock series presented in Perez Toledo (2003).

Table 4. Calibrated Parameters for Chile

A	A^H	α	β	δ	δ_H	g	R^*	ϕ	$\bar{b}$	σ	ρ
1	0.1437	0.61	0.6	0.02	0	0.12	0.04	1	-0.34	1.6	0.96

Source: Authors' calculations.

The variables ϕ and $\bar{b}$ that appear in the table belong to the particular specification that was used of the function $R\left(.\right)$, taking the form:

$$R\left(\frac{B^p + B^g}{K}\right) = R^* + \phi\left[e^{\left\{\bar{b} - \frac{B^p+B^g}{K}\right\}} - 1\right],$$

where R^* is the international interest rate. This is the same function that appears in Schmitt-Grohe and Uribe (2003).

We chose the value of ϕ to be the smallest value that is consistent with having closed the economy. For $\bar{b}$, we used the average of the Net International Investment Position to GDP ratio from 1997 to 2008. Finally, for R^*, we used the average 1-year treasury bill rate, as we needed some measure of the international interest rate that the economy faced on an annual basis (we calibrated the parameters to match this interpretation). We chose ρ to validate the statement $\rho\left(1 + R^*\right) = 1$, which is standard in literature on small open economies. Finally, we took the value of σ from Arrau (1990), and chose the value of η to be one.