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The Role of Financial Frictions

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Abstract

[1] Countercyclical country interest rates have been shown to be both a distinctive characteristic and an important driving force of business cycles in emerging market economies. In order to account for this, most business cycle models of emerging market economies have relied on ad hoc and exogenous countercyclical interest rate processes. This paper embeds a financial contract à la Bernanke et al. (1999) in a standard small open economy business cycle model that endogenously delivers countercyclical interest rates. The model is then applied to the data, drawn from a novel panel dataset for emerging economies that includes financial data, namely sovereign and corporate interest rates as well as leverage. It is shown that the model accounts well not only for countercyclical interest rates, but also for other stylized facts of emerging economies' business cycles, including the dynamics of leverage.

JEL classifications: E32, E44, F41

Keywords: Business cycle models, Emerging economies, Financial frictions

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1 Introduction

A well-documented stylized fact in international macroeconomics is the significant difference in business cycles between emerging and developed economies. Relative to advanced economies, fluctuations in emerging markets are continuously characterized by large volatility in output and even higher volatility in consumption and investment, which leads to countercyclical dynamics of the trade balance.[2] Another key difference lies in the cyclicality of borrowing costs faced in international financial markets. While in emerging economies real interest rates are strongly countercyclical and volatile, in developed economies they are mildly procyclical and less variable.

In this paper we focus on amplification mechanisms that provide a microfounded rationale for interest rate dynamics in emerging economies. In particular, we analyze frictions that may arise in the market for private debt due to asymmetric information and moral hazard. We also argue that the dynamics of interest rates cannot be fully understood in disconnect from entrepreneurial borrowing. Therefore we start our analysis by constructing a novel dataset on leverage of corporate, non-financial firms in emerging countries and providing evidence on its dynamics over the cycle. We extend the datatset with updated series from national income accounts as well as sovereign and corporate interest rates. We find that assets-to-equity ratios are countercyclical in the data and that leverage dynamics are strikingly similar to those of interest rates. We also corroborate the stylized facts documented in the literature and show that they are robust to the inclusion of the recent financial crisis episode.

In order to account for these empirical facts, we build a business cycle model where domestic interest rates are fully endogenous and determined by the default risk in the private sector. We do so by embedding a financial contract à la Bernanke et al. (1999), henceforth BGG, into an otherwise standard real business cycle model of a small open economy in which a productivity shock is the sole driving force. This financial structure also allows for endogenous fluctuations of leverage. The interest rate premium stems endogenously from agency problems between foreign lenders and domestic borrowers. The financial contract is then designed to minimize the expected agency costs in an environment where corporate default occurs in equilibrium. We focus on the propagation role of the financial accelerator in accounting for the stylized facts, especially the dynamics of interest rates and leverage. We argue that this mechanism is well suited for accounting for the data patterns in emerging economies, because it naturally gives rise to countercyclical interest rates and leverage akin to those observed in the data. For example, a positive productivity shock not only increases output, but also increases the net worth of entrepreneurs, thereby reducing leverage as well as the aggregate default risk and hence lowering the country premium.

[2] See the works by Agénor et al. (2000), Aguiar and Gopinath (2007a) and Benczúr and Rátfai (2010).

We apply our model to emerging economies' data and estimate the parameters governing the financial contract as well as the productivity process. We do so by matching some of the key second moments that distinguish emerging economies from their developed counterparts. To do so we use a panel of countries from our dataset. In that sense, another contribution of the paper lies in using a more comprehensive set of emerging economies instead of focusing on a single country as has been previously done.

The main findings of the estimation exercise can be summarized as follows. The financial structure of our model allows us to properly account for the dynamics of emerging economies' business cycles. Most importantly, it endogenously generates a strong volatility and countercyclicality of interest rates, similar to their empirical counterparts. The results indicate that, through the lens of our model, the data is seen as characterized by relatively high levels of steady-state leverage. This leverage allows the model to generate large movements in entrepreneurial net worth and, in consequence, in the country risk premium. The intuition behind this is simple. Following a positive productivity shock, a leveraged entrepreneur in an emerging economy will experience very high profits, increase equity and optimally deleverage on the margin. On aggregate such behavior will imply that leverage and income move in opposite directions. Therefore, the model also accounts for the countercyclicality of the leverage itself and replicates closely the dynamics observed in the data. Based on these findings we argue that leverage has an important role in accounting for both the large volatility and countercyclicality of interest rates in emerging economies. Hence, we contribute to the literature by providing a model that rationalizes such dynamics.

Our work is a continuation of the research program on business cycles in emerging economies. Since at least the work of Agénor et al. (2000), the key differences in aggregate dynamics between developing and advanced economies have been well known. Later works by Neumeyer and Perri (2005) and Uribe and Yue (2006) also stressed the countercyclicality of interest rates in these countries. Motivated by those stylized facts, these works built business cycle models in which exogenous interest rate shocks are the main driving force and reduced-form frictions act as a powerful amplification mechanism for standard productivity shocks. Such frictions take the form of working capital requirements and country-specific spreads that react to country fundamentals. Neumeyer and Perri (2005) and Uribe and Yue (2006), as well as Oviedo (2005), find that these frictions are a key component in order for their models to successfully account for the observed business cycle facts. In Aguiar and Gopinath (2007b) it is shown that a business cycle model in which the country interest rate is not orthogonal to productivity shocks does well in matching the features of the data in emerging market countries. The relevance of spreads linked to fundamentals has also been stressed by Chang and Fernández (2010) when accounting for the Mexican business cycle, in particular during the Tequila Crisis episode. Lastly, García-Cicco et al. (2010) have shown that a high elasticity of the interest rate premia to debt levels is needed to mimic trade balance dynamics in Argentina.

Up to that point, however, the literature has been silent about why the country premium would depend upon domestic variables such as output or the productivity level. In doing so the literature has taken as given, rather than derived from first principles, the laws of motion for the country interest rates. Arellano (2008) contributes to the understanding of country premia by providing a theoretical framework for the link between country spreads and fundamentals within a model of strategic sovereign default. In her model, sovereign default probabilities are high when expectations of productivity are low. Hence this study provides a theoretical framework for the link between country spreads and fundamentals although within an endowment economy. This framework has recently been extended by Mendoza and Yue (2012), who also study sovereign default in a production economy. While these papers advance in simultaneously endogenizing default risk and aggregate fluctuations, this line of research focuses exclusively on sovereign risk. However, virtually no study has jointly assessed quantitatively the relationship between corporate default, business cycles and emerging markets' interest rates within a dynamic general equilibrium framework. We think that gap in the literature is an important one because high business cycle volatility characterizes several emerging economies which have experienced neither sovereign default nor serious fiscal solvency concerns within the time intervals under study. Our work aims to fill this gap in the literature.

Aside from the research discussed above, our work can also be associated with other studies in the literature. In an important work, Céspedes et al. (2004) use the framework developed by BGG to stress the role of balance sheet effects during the financial crises that affected developing economies in the 1990s. In their framework liabilities are dollarized and hence exchange rate movements lead to fluctuations in net worth. This framework was later enriched by including, among other things, a nontradable sector (Devereux et al. (2006)) and endogenous capital utilization rates (Gertler et al. (2007)). Elekdağ et al. (2006) develop a model in a similar spirit and estimate it using Bayesian methods.[3] Overall, the objective of these works has been to evaluate the strength of balance sheet effects. Yet, neither assesses the implications of financial frictions for country interest rates along the business cycle, which is central to our analysis. Recently, Akinci (2011) has embedded a financial contract á la BGG into a business cycle model for Argentina, but unlike here, the main driving force in her model is a risk shock similar to that of Christiano et al. (2010).

Lastly, our work can be linked to studies which stress the importance of the financial accelerator in developed economies. The works by Iacoviello (2005), Christiano et al. (2010) and Fuentes-Albero (2012),

[3] Further important research on the role of credit market imperfections in emerging economies has been made by Tornell and Westermann (2005), who present empirical evidence on the presence of financial frictions in middle income countries in the form of sectoral asymmetries in financing opportunities. They show that smaller firms from nontradable sectors have less access to credit than larger companies in tradable sectors. They also point to low levels of contract enforceability and high levels of currency mismatch that result from liabilities being denominated in foreign currencies. This mismatch manifests itself in the balance sheets of banks and leads to a high exposure of the nontradable sector.

among others, have greatly enriched the understanding of the relevance of financial frictions for business cycle fluctuations in developed economies. Clearly, our work extends this research agenda to emerging economies.

This paper is divided into seven sections including this introduction. In section 2 we first review the previous empirical contributions to the analysis of business cycles in emerging economies and then we compare them to the stylized facts from our dataset. Section 3 presents our business cycle model of a small open economy. Section 4 summarizes our estimation strategy. The results of the paper are then presented in section 5. In section 6 we discuss the key leverage mechanism which is at work in our model and which drives our results. Concluding remarks are given in section 7. Appendices A through E gather some technical details of our analysis.

2 Stylized Facts in Emerging Market Business Cycles

Since at least the study by Agénor et al. (2000) it has been documented that some of the business cycle patterns in emerging economies differ in nontrivial ways from those observed in developed ones. Business cycle characteristics have been traditionally studied by computing simple unconditional second moments of filtered quarterly macroeconomic series. They concluded that fluctuations of output, investment and consumption in emerging economies are all more volatile than the corresponding series in advanced economies and that consumption in the former is more volatile than output itself, thereby generating strong countercyclicality in the trade balance. These stylized facts were confirmed by Neumeyer and Perri (2005), who constructed a dataset of five emerging and five developed economies. These authors also noted that real country interest rates in emerging economies are strongly countercyclical and tend to lead the cycle.[4] Interest rates in developed economies, on the other hand, are, if anything, procyclical. Aguiar and Gopinath (2007a) extended the sample of analysis to 13 small open economies in each of the two groups of countries, between the 1980s and 2003. Their results are in line with previous studies. Importantly, however, they did not document interest rates dynamics.[5] Also, none of these papers explicitly looked at the dynamics of leverage.

In this section we present updated evidence on business cycle characteristics of emerging countries. Our dataset has been initially constructed by Fernández and Zamora (2011). Relative to Aguiar and Gopinath (2007a), this dataset is extended in three dimensions. First, all series have been updated until 3Q 2010. This means an extension of seven years which allows us to assess whether the existing stylized facts are robust to

[4] Similar results were obtained in Uribe and Yue (2006) using a larger pool of emerging economies.

[5] Benczúr and Rátfai (2010) undertook a similar analysis for the former Communist Bloc countries. They found some heterogeneity across these economies. In particular, business cycles in Bulgaria, Romania and Russia tend to be more volatile and resemble emerging economies in other regions, whereas cycles in the Visegrád Group countries (Czech Republic, Hungary, Poland, Slovakia) tend to be broadly in line with those for developed economies.

the inclusion of the 2007-2009 financial crisis period. Second, it is extended with information on real country interest rates, in the spirit of Neumeyer and Perri (2005) and Uribe and Yue (2006).[6] Finally, we provide information on leverage.

Tables 1– 3 present some of the key unconditional second moments that characterize business cycles across emerging market economies. While table 1 shows averages across countries and compares them to developed economies[7], tables 2 and 3 zoom in on country-specific moments. Aggregate volatility, measured by percentage deviation of GDP[8] from its Hodrick-Prescott (HP) trend, is almost twice as large in emerging markets as in developed ones. The relative volatilities of the two largest components of aggregate demand, consumption and investment, are also roughly 50% larger in the former group than in the latter. In eight out of thirteen emerging countries the volatility of consumption is actually higher than that of output. Only two developed economies exhibit this property. Correlations of both consumption and investment with output are nonetheless quite similar across the two pools of economies. In consequence, emerging economies exhibit much more volatile and countercyclical trade balances (12 cases) than developed ones (4 cases).

We now turn our attention to real interest rates. In emerging economies they include relatively large country-specific spread components. These spreads have been frequently proxied in the literature using the Emerging Markets Bond Index (EMBI), which is based on sovereign bonds. As can be seen in Table 1, real, EMBI-based interest rates in emerging economies tend to be countercyclical,[9] as indicated by the statistically significant correlation coefficient value of -0.35.[10] This is in contrast to the number for developed economies,

[6]While it was possible to update the data for all 13 developed economies that Aguiar and Gopinath (2007a) considered, lack of data on interest rates prevented us from doing so for Israel and the Slovak Republic or from extending the pool to other Eastern European countries. Also, in light of the results of Benczúr and Rátfai (2010), we decided instead to provide a more comprehensive picture for Latin America by including data on Chile and Colombia in the set.

[7]Our set of developed economies is the one considered by Aguiar and Gopinath (2007a): Australia, Austria, Belgium, Canada, Denmark, Finland, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden and Switzerland. Our list of emerging economies is presented in table 2.

[8]To be consistent with the model presented in the next section, our measure of GDP does not incorporate government spending. See appendix A for further details regarding the dataset.

[9]Following Neumeyer and Perri (2005) we measure real country interest rates in emerging market economies as product between the 3-Month U.S. real T-Bill rate and the country-specific EMBI. For developed economies we use interest rates from 90-day corporate commercial paper or interbank rates and deflate them by subtracting the expected GDP deflator (or, when not available, the CPI) inflation from the nominal rate. Real U.S. rates are obtained by subtracting the expected CPI inflation from the nominal rate. Expected inflation is computed as the average of inflation in the current and the three preceding months. See Appendix A for further details.

[10]This number is very similar (-0.30) if one uses sovereign credit default swap spreads, another commonly used proxy for risk.

Table 1: Emerging and developed markets' business cycle moments.

Second moment[a]	**Emerging markets**		**Developed markets**	
$\sigma(Y)^b$	3.62^c	(0.27)	1.70	(0.20)
$\sigma(C)/\sigma(Y)$	1.09	(0.07)	0.64	(0.02)
$\sigma(I)/\sigma(Y)$	3.41	(0.28)	2.45	(0.11)
$\sigma(TB)$	3.06	(0.30)	1.29	(0.09)
$\rho(TB,Y)$	−0.30	(0.07)	0.32	(0.04)
$\rho(C,Y)$	0.73	(0.05)	0.59	(0.05)
$\rho(I,Y)$	0.71	(0.04)	0.64	(0.06)
$\sigma(R)$	3.57	(0.21)	3.20	(0.83)
$\rho(R,Y)$	−0.35	(0.06)	0.01	(0.04)

[a] All series were logged (except for TB), and then HP filtered. The GMM estimated moments are computed as weighted averages, i.e. based on unbalanced panels.

[b] σ denotes standard deviation, ρ denotes correlation coefficient.

[c] Standard deviations are expressed in %. Standard errors are reported in brackets.

0.01, which indicates interest rate acyclicality. Interest rates are also relatively more volatile in the former group of countries than in the latter.[11]

Importantly, the strong volatility and countercyclicality of interest rates is robust to non-sovereign measures of risk, for example the corporate emerging market bond index (CEMBI) spreads. Table 4 documents that virtually all emerging countries exhibit a very high comovement between sovereign and corporate measures of risk, with correlations between EMBI and CEMBI spreads ranging between 0.80 and 0.99. This strong comovement provides an empirical validation for our model to be presented below in which risk premia emanate from default in the private rather than the public sector. Furthermore, in table 5 we assess the cyclicality of country interest rates using CEMBI spreads. While data limitations prevent us from conducting the analysis for the whole sample of emerging economies, one can easily observe that with this measure of corporate risk the countercyclicality and volatility of interest rates are even higher. The correlation coefficient is now −0.61 as opposed to −0.52 when EMBI was used, whereas the standard deviation increases from

[11] This last difference is to a large degree driven by Argentina and Ecuador (see table 2), although it remains statistically significant even when they are removed from the sample. Argentina and Ecuador are the only countries that experienced sovereign default within the timeframe of our dataset (see Reinhart and Rogoff (2009)). However, these two countries do not affect the interest rate countercyclicality observed in emerging economies, as can be seen in Table 3.

Table 2: Volatility of main macro variables in emerging economies.

Emerging Economy[a]	$\sigma(Y)^b$		$\sigma(C)/\sigma(Y)$		$\sigma(I)/\sigma(Y)$		$\sigma(TB)$		$\sigma(R)$	
Argentina	4.90[c]	(0.70)	1.24	(0.11)	3.34	(0.51)	3.11	(0.81)	8.78	(1.27)
Brazil	2.36	(0.36)	0.93	(0.13)	3.66	(0.65)	1.04	(0.15)	2.40	(0.40)
Chile	2.03	(0.28)	1.26	(0.20)	4.35	(0.52)	3.52	(0.66)	1.18	(0.20)
Colombia	2.53	(0.52)	0.96	(0.09)	4.53	(0.42)	1.80	(0.39)	1.50	(0.14)
Ecuador	2.33	(0.34)	1.11	(0.10)	7.38	(1.03)	4.62	(0.84)	6.41	(1.22)
Korea	3.66	(0.75)	1.49	(0.09)	3.35	(0.24)	4.18	(1.05)	1.23	(0.30)
Malaysia	3.07	(0.45)	1.47	(0.23)	6.09	(0.40)	5.07	(1.01)	1.57	(0.24)
Mexico	2.88	(0.34)	1.28	(0.19)	2.85	(0.25)	1.59	(0.40)	2.20	(0.47)
Peru	2.20	(0.34)	1.17	(0.16)	4.39	(0.32)	2.14	(0.38)	1.35	(0.15)
Philippines	2.94	(0.43)	0.57	(0.13)	2.26	(0.46)	3.69	(0.61)	1.24	(0.17)
South Africa	1.86	(0.33)	0.91	(0.07)	3.39	(0.38)	1.33	(0.09)	1.28	(0.15)
Thailand	3.27	(0.77)	1.05	(0.10)	4.37	(0.34)	4.22	(0.98)	1.07	(0.21)
Turkey	6.85	(0.74)	0.63	(0.07)	2.43	(0.25)	1.58	(0.28)	1.46	(0.11)

[a] All series were logged (except for TB), and then HP filtered. Moments and their corresponding standard errors were computed using GMM.

[b] σ denotes standard deviation.

[c] Standard deviations are expressed in %. Standard errors are reported in brackets.

1.37% to 1.68%.

The final empirical exercise we perform is an analysis of leverage dynamics in emerging economies. It serves several purposes. First, we think that it is a natural follow-up of the analysis of interest rate and risk spread movements. Leverage plays a key role in many macroeconomic models of financial frictions with endogenous risk premia. For example, leverage measured as assets-to-equity ratio enters as an argument in the loan supply curve in Bernanke et al. (1999). Secondly, we are not aware of any other study performing such an analysis. The closest work is that of Mendoza and Terrones (2008), who study episodes of credit booms in emerging economies. In their paper leverage goes up with GDP. Yet, these authors specifically condition the data on rapid credit buildup episodes. We are instead interested in unconditional leverage fluctuations over the whole cycle. Developed countries have received slightly more attention, for example by Chugh (2011). That study reports strong countercyclical debt-to-equity ratios in the United States, when measured at book values.

Finance literature distinguishes between several measures of firm leverage, potentially varying in properties

Table 3: Correlations of main macro variables with output in emerging economies.

Emerging Economy[a]	$\rho\,(TB, Y)$[b]		$\rho\,(C, Y)$		$\rho\,(I, Y)$		$\rho\,(R, Y)$	
Argentina	−0.65	(0.09)[c]	0.91	(0.02)	0.83	(0.07)	−0.56	(0.12)
Brazil	−0.05	(0.18)	0.70	(0.11)	0.62	(0.15)	−0.40	(0.14)
Chile	0.29	(0.20)	−0.14	(0.20)	0.49	(0.16)	−0.04	(0.29)
Colombia	−0.62	(0.16)	0.85	(0.07)	0.82	(0.08)	−0.22	(0.14)
Ecuador	−0.48	(0.23)	0.78	(0.10)	0.71	(0.14)	−0.48	(0.15)
Korea	−0.84	(0.07)	0.91	(0.05)	0.95	(0.02)	−0.66	(0.16)
Malaysia	−0.48	(0.19)	0.43	(0.23)	0.78	(0.07)	−0.52	(0.18)
Mexico	−0.58	(0.15)	0.75	(0.06)	0.81	(0.07)	−0.48	(0.16)
Peru	−0.25	(0.17)	0.30	(0.13)	0.89	(0.04)	−0.33	(0.22)
Philippines	0.81	(0.09)	−0.02	(0.16)	0.02	(0.19)	0.19	(0.15)
South Africa	−0.27	(0.20)	0.83	(0.07)	0.73	(0.12)	0.06	(0.23)
Thailand	−0.43	(0.20)	0.81	(0.05)	0.75	(0.12)	−0.58	(0.08)
Turkey	−0.04	(0.18)	0.72	(0.06)	0.87	(0.04)	−0.40	(0.14)

[a] All series were logged (except for TB), and then HP filtered. Moments and their corresponding standard errors were computed using GMM.

[b] ρ denotes correlation coefficient.

[c] Standard errors are reported in brackets.

and dynamics. In this paper we focus on the assets-to-equity ratio[12]. The ratio can be computed either using historical (book) or market values. We use market value of equity, which is proxied by total market capitalization of firms. The data is readily available for publicly traded firms. On the other hand, we use book value of debt. The reason for this is twofold. First, it is the book value of debt that has to be returned to the lender, rather than the market value, as pointed out by Levin et al. (2006). Secondly, trade in corporate debt is rare, except for the largest firms, and frequently illiquid, not least in emerging economies, so no reliable data is available.

Firm-level data of quarterly frequency are taken from Bloomberg. It encompasses the emerging countries

[12] Geanakoplos (2010) makes the case for the loan-to-value ratio as a more appropriate indicator. Being a flow, it reflects the business cycle phase more quickly. Also, stock market crashes may potentially generate a perverse result of an increase of the assets-to-equity ratio. However, in our leverage data, we observe, as expected, an unfolding deleveraging just before a recession hits. In fact, such deleveraging occurs despite the fact that debt is measured at book value and equity at market value, which makes the ratio potentially prone to perverse leverage hikes. Finally, if refinancing is possible, the distinction between loan-to-value and debt-to-assets can largely fade.

Table 4: Correlations between *EMBI* and *CEMBI* spreads in emerging economies.

Country[a]	**Period**	$\rho(EMBI, CEMBI)^b$
Brazil	4Q 2003 - 4Q 2011	0.96 (0.02)[c]
Chile	3Q 2009 - 4Q 2011	0.99 (0.00)
Colombia	4Q 2007 - 4Q 2011	0.96 (0.01)
Korea	3Q 2002 - 2Q 2004	0.98 (0.06)
Malaysia	4Q 2001 - 4Q 2011	0.98 (0.00)
Mexico	4Q 2001 - 4Q 2011	0.80 (0.06)
Peru	3Q 2005 - 4Q 2011	0.95 (0.01)
Philippines	4Q 2009 - 3Q 2010	0.99 (0.00)
South Africa	4Q 2009 - 4Q 2011	0.82 (0.10)
Turkey	3Q 2010 - 4Q 2011	0.88 (0.05)

[a] All series were logged and then HP filtered. Moments and their corresponding standard errors were computed using GMM.

[b] ρ denotes correlation coefficient.

[c] Standard errors are reported in brackets.

Table 5: *EMBI* and *CEMBI*-based interest rate moments in emerging economies.

Country[a]	**Period**	$\sigma(Y)^b$	$\sigma(R_{EMBI})$	$\sigma(R_{CEMBI})$	$\rho(Y, R_{EMBI})$	$\rho(Y, R_{CEMBI})$
Brazil	4Q 2003 - 3Q 2010	2.07[c] (0.51)	1.31 (0.28)	1.41 (0.27)	−0.77 (0.12)	−0.77 (0.13)
Malaysia	4Q 2001 - 3Q 2010	2.30 (0.63)	1.32 (0.21)	1.35 (0.20)	−0.48 (0.20)	−0.54 (0.21)
Mexico	4Q 2001 - 3Q 2010	2.56 (0.54)	1.36 (0.21)	2.04 (0.54)	−0.35 (0.31)	−0.61 (0.18)
Peru	3Q 2005 - 3Q 2010	2.64 (0.47)	1.55 (0.28)	1.80 (0.39)	−0.61 (0.21)	−0.59 (0.17)
All	4Q 2001 - 3Q 2010	2.39 (0.29)	1.37 (0.12)	1.68 (0.23)	−0.52 (0.13)	−0.61 (0.10)

[a] All series were logged and then HP filtered. Moments and their corresponding standard errors were computed using GMM.

[b] σ denotes standard deviation, ρ denotes correlation coefficient.

[c] Standard deviations are expressed in %. Standard errors are reported in brackets.

described before except for Ecuador, where no data were available.[13] We focus solely on firms from corporate,

[13] The available leverage series are shorter than those used in atables 1 through 3 and the model estimation. For instance, leverage data do not include data for Korea during the East Asian crisis. See Appendix A for details on the number of firms used in the computation of leverage.

non-financial sectors. We do so because the leverage in the model describes the asset structure of entrepreneurs in the production sector, not of lenders. Also, financial firms tend to hold sovereign debt in their assets. Therefore, their leverage and interest rate dynamics is directly influenced by the performance of the sovereign. The data were additionally filtered for outliers. The average market leverage ratio for a given country in a given year was computed using total market capitalization as weights. Lastly, the series were also HP filtered. Technical details are provided in Appendix A.

Leverage dynamics over the business cycle are reported in Figure 1. It documents serial correlations of leverage with output, i.e. $Corr(Y_t, Lev_{t+j})$, where j is measured on the X-axis. The first important message is that the assets-to-equity ratio is, on average, countercyclical in the data.[14] Contemporaneous correlation with the cycle is -0.31 on average and becomes most negative between $j = -2$ and $j = 0$. The precise location of the peak depends on data filtering. Therefore it is not clear whether the variable leads or is concurrent with the cycle. A clear shift to positive correlation occurs only for $j = 2$. To illustrate this with an example, suppose that output is below its trend in $j = 0$. According to the figure, one should expect leverage to have been above its long-run mean during the previous three periods. Deleveraging starts only at $j = 0$ and lasts for the following periods.

Another important pattern emerges when one compare the cyclicalities of leverage and interest rates. The right panel of figure 1 is a version of Figure 3(a) from Neumeyer and Perri (2005), reconstructed using our extended and updated dataset. The pattern of interest rate dynamics is qualitatively the same as that for leverage, except for slight differences for $j \leq -3$. Finally, relative to Neumeyer and Perri (2005), we do not find clear support for the statement that interest rates lead the cycle, as our trough occurs at $j = 0$. Similarly as with leverage, the exact minimum is sensitive to the data sample at hand and the filtering method applied.

Summing up, we find that business cycles in emerging economies continue to exhibit patterns that are different than those observed in developed countries despite the large worldwide macroeconomic volatility observed during the recent financial turmoil. The former group continues to display countercyclical real interest rates, regardless of whether they are measured using sovereign or corporate risk measures. Therefore, our results broadly corroborate the findings of previous studies.

Importantly, we extend the research agenda to the analysis of leverage, an issue not studied before in this context. We provide primary evidence on unconditional leverage dynamics over the business cycle in emerging economies. In particular, we find patterns of leverage countercyclicality in most countries in the sample. Moreover, its dynamics are very similar to those of interest rate premia. These findings suggest an

[14] Technically, countercyclicality of leverage occurs here, because (CPI-deflated) equity is procyclical (correlation with output is 0.25), whereas debt is essentially acyclical (correlation with output is 0.00). Both equity and debt are an order of magnitude more volatile than output, with standard deviation equal to 0.37 in both cases. Correlation of equity with debt reaches 0.51.

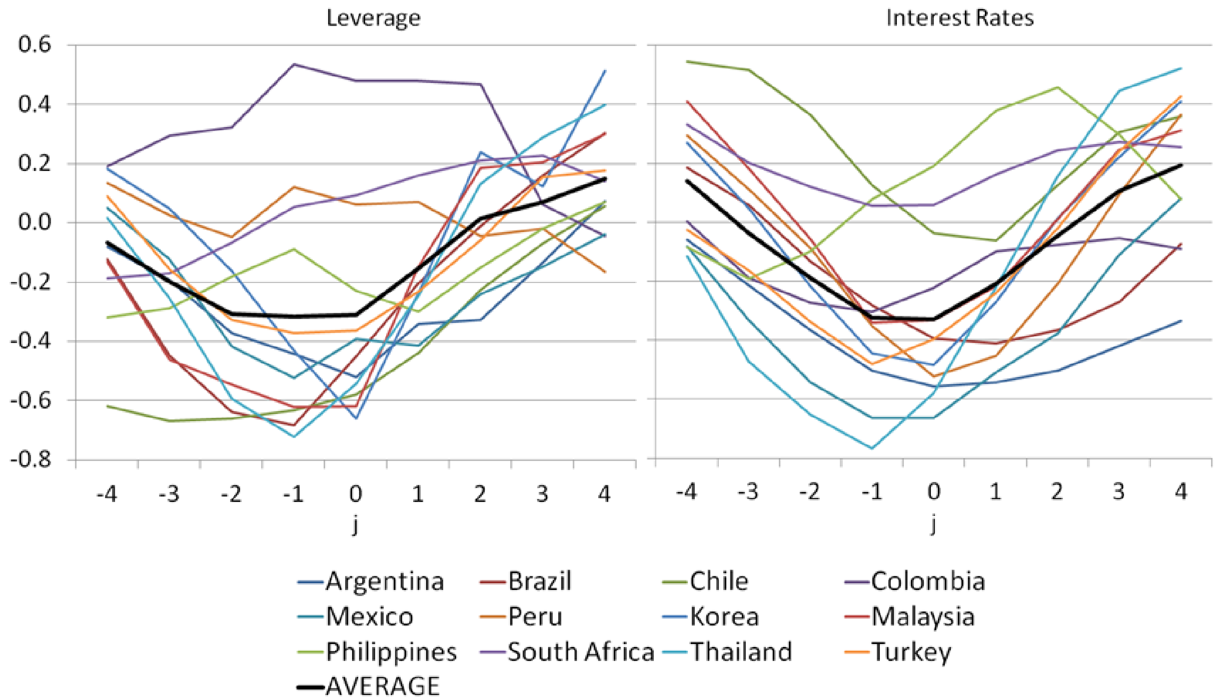


Figure 1: Leverage and interest rate cyclicality in emerging economies: $Corr\left(Y_t, Lev_{t+j}\right)$ (left panel) and $Corr\left(Y_t, R_{t+j}\right)$ (right panel). Leverage is computed as assets-to-equity ratio at market values. Interest rates are real U.S. T-bill rates plus country-specific EMBI.

important role for a financial accelerator mechanism by which interest rate premia are linked to leverage. Thus, in the next section we embed that mechanism in a business cycle model of a small open economy in which interest rates are endogenously determined and driven by fluctuations of leverage.

3 Model

Our starting point is a real business cycle model of a small open economy (see, e.g., Mendoza (1991)). A key modification is to extend it with a financial accelerator mechanism, developed by Carlstrom and Fuerst (1997) and Bernanke et al. (1999). We follow the latter exposition and describe it in detail in subsection 3.1. The model economy is inhabited by four types of agents: households, entrepreneurs, capital producers, and a foreign sector which is the only source of credit for the domestic economy.

3.1 Entrepreneurs

In this framework, the key role is played by entrepreneurs. The sector is perfectly competitive and produces a homogenous final good which is later consumed or used for investment. At the heart of the financial accelerator mechanism is the fact that entrepreneurs have to borrow funds from lenders in order to finance their production, in particular to purchase capital from capital-producing firms. Therefore, the assets of an i-th entrepreneur are the sum of her net worth $\tilde{N}_{i,t+1}$ and borrowed funds $\tilde{B}_{i,t+1}$:

$$Q_t \tilde{K}_{i,t+1} = \tilde{N}_{i,t+1} + \tilde{B}_{i,t+1} \tag{3.1}$$

where $\tilde{K}_{i,t+1}$ is the capital stock and Q_t is the price of capital expressed in terms of final goods.[15] We assume that all borrowing takes place from abroad. The production function of an i-th entrepreneur is given by

$$\tilde{Y}_{i,t} = \omega_{i,t} A_t \tilde{K}_{i,t}^{\alpha} \left(\tilde{X}_t L_{i,t} \right)^{1-\alpha}$$

where $L_{i,t}$ is labor input, respectively and A_t is the economy-wide level of total factor productivity, which follows a stationary stochastic process:

$$\ln A_t = \rho_A \ln A_{t-1} + (1 - \rho_A) \ln A + \epsilon_{A,t}, \quad |\rho_A| < 1 \tag{3.2}$$

where $\epsilon_{A,t} \overset{i.i.d}{\sim} (0, \sigma_A)$.

Additionally, every entrepreneur is subject to an idiosyncratic productivity shock captured by $\omega_{i,t}$. In each period the realization of the shock is random and comes from a log-normal distribution with expected value $E\omega_{i,t} = 1$. It is assumed that the realization of $\omega_{i,t}$ is private information of the entrepreneur. In order to learn this value, the foreign lender has to pay a monitoring cost μ, which is a fraction of the entrepreneur's remaining assets (output plus undepreciated capital).[16] The optimal contract between (foreign) lenders and (domestic) entrepreneurs specifies a cutoff value of ω_t, denoted as $\bar{\omega}_t$.[17] Entrepreneurs whose realized $\omega_{i,t}$ falls below $\bar{\omega}_t$ are considered bankrupt and monitored, and their estates are taken over by lenders. The net income of lenders from bankrupt entrepreneurs is therefore

$$(1 - \mu) \int_0^{\bar{\omega}_t} \omega_{i,t} f(\omega_{i,t}) \, \mathrm{d}\omega_{i,t} \, R_{i,t}^K Q_{t-1} \tilde{K}_{i,t}$$

where

$$R_{i,t}^K = \frac{\alpha \frac{\tilde{Y}_{i,t}}{\tilde{K}_{i,t}} + Q_t (1 - \delta)}{Q_{t-1}} \tag{3.3}$$

[15] The model economy is assumed to follow a deterministic trend $\tilde{X}$ with the growth rate $\frac{\tilde{X}_{t+1}}{\tilde{X}_t} = g \geq 1$. We use tildes to denote variables that trend in equilibrium, e.g. $\tilde{K}_t = K_t \tilde{X}_t$.

[16] The financial contract with asymmetric information and agency costs is based on the idea initially developed by Townsend (1979).

[17] Note that the optimal contract is homogenous and standardized across entrepreneurs. Also, there exists one aggregated loan supply curve, identical for all entrepreneurs. This aggregation, a complex problem in principle, is possible because of a few assumptions introduced to the model, in particular constant returns to scale of the entrepreneurial production function, independence of $\omega_{i,t}$ from history as well as the constant number of entrepreneurs in the economy and their risk neutrality and perfect competitiveness. See Carlstrom and Fuerst (1997) or Bernanke et al. (1999) for a more detailed dicussion.

is the ex post return on capital. Optimality implies that firms with $\omega_{i,t} \geq \bar{\omega}_t$ will pay their debts, retain the profit and will not be monitored. The revenue of lenders from solvent entrepreneurs is

$$\bar{\omega}_t \int_{\bar{\omega}_t}^{\infty} f(\omega_{i,t}) \, \mathrm{d}\omega_{i,t} \, R_{i,t}^K Q_{t-1} \tilde{K}_{i,t}$$

The timing of events is as follows. At the end of $t-1$, there is a pool of entrepreneurs, whose equity is $\tilde{N}_t$. Those firms decide upon the optimal demanded level of capital $\tilde{K}_t$, and hence the level of borrowing $\tilde{B}_t$. At this point R_t^K is not known, since time t TFP shock has not yet realized. However, the riskless international rate R^* over which the risk premium is determined (i.e., the rate from $t-1$ until t) is known. The cutoff value for the optimal contract $\bar{\omega}_t$ is not yet determined, so entrepreneurs make their decision based upon $E_{t-1}\bar{\omega}_{i,t}$, subject to the zero-profit condition of the lenders. Formally, they solve the following profit-maximization problem:

$$\max_{\tilde{K}_{i,t}, E_{t-1}\bar{\omega}_t} E_{t-1} \left[1 - \Gamma(\bar{\omega}_t)\right] R_{i,t}^K Q_{t-1} \tilde{K}_{i,t}$$

subject to

$$R^* \left(Q_{t-1} \tilde{K}_{i,t} - \tilde{N}_{i,t}\right) = \left[\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)\right] R_{i,t}^K Q_{t-1} \tilde{K}_{i,t} \tag{3.4}$$

where

$$\Gamma(\bar{\omega}_t) \equiv \int_0^{\bar{\omega}_t} \omega_{i,t} f(\omega_{i,t}) \, \mathrm{d}\omega_{i,t} + \bar{\omega}_t \int_{\bar{\omega}_t}^{\infty} f(\omega_{i,t}) \, \mathrm{d}\omega_{i,t} \quad \text{and} \quad G(\bar{\omega}_t) \equiv \int_0^{\bar{\omega}_t} \omega_{i,t} f(\omega_{i,t}) \, \mathrm{d}\omega_{i,t}$$

The left-hand side of the optimization constraint expresses the opportunity cost of lending, i.e., the gross return on a riskless loan. The right-hand side expresses lenders' returns on a risky loan net of monitoring costs. It includes the repayment from solvent borrowers (a fraction given by the second component of $\Gamma(\bar{\omega}_t)$), as well as the bankrupt's estate (i.e., first component of fraction $\Gamma(\bar{\omega}_t)$), net of monitoring costs $\mu G(\bar{\omega}_t)$. The combined first order conditions yield[18]

$$E_{t-1} \left\{ \left[1 - \Gamma(\bar{\omega}_t)\right] \frac{R_t^K}{R^*} + \frac{\Gamma_{\bar{\omega}}(\bar{\omega}_t)}{\Gamma_{\bar{\omega}}(\bar{\omega}_t) - \mu G_{\bar{\omega}}(\bar{\omega}_t)} \left[\frac{R_t^K}{R^*} \left(\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)\right) - 1 \right] \right\} = 0 \tag{3.5}$$

Next, the morning of t comes and the aggregate TFP shock is realized. Its value pins down the aggregate return on capital R_t^K as well as the other non-predetermined variables. The value of $\bar{\omega}_t$, i.e., the threshold which determines the bankruptcy cutoff is pinned down. Since lenders are perfectly competitive, $\bar{\omega}_t$ simply solves the zero-profit condition 3.4, where the i index has been dropped because of aggregation (see footnote 17). Once $\bar{\omega}_t$ is set and the idiosyncratic productivity shock is realized, some firms go bust, while others remain solvent. However, this is important only at the firm level. On the aggregate level the economy-wide

[18] Second-order conditions which guarantee a maximum are provided in Appendix B. This equation constitutes, after some modifications, a basis of the entrepreneurial demand for capital. See Christiano et al. (2010) for a detailed discussion.

rate of return R_t^K and output $\tilde{Y}_t$ had already been known when the aggregate shock was realized, i.e., at the dawn of t.

We also assume that a fraction of entrepreneurial profit $1-\phi$ is paid out as dividends and consumed every period. Therefore, $1-\phi$ can be interpreted as a dividend payout ratio and shareholders' consumption is expressed as:

$$\tilde{C}_t^e = (1-\phi)\,\tilde{V}_t \tag{3.6}$$

where

$$\tilde{V}_t = R_t^K Q_{t-1}\tilde{K}_t - \left(R^* + \frac{\mu\int_0^{\bar{\omega}_t}\omega f(\omega)\,\mathrm{d}\omega\, R_t^K Q_{t-1}\tilde{K}_t}{Q_{t-1}\tilde{K}_t - \tilde{N}_t}\right)\left(Q_{t-1}\tilde{K}_t - \tilde{N}_t\right) \tag{3.7}$$

and $\tilde{V}_t$ is the aggregate ex post value of entrepreneurial firms[19] computed as the gross return on their capital (first term) less debts of the solvent firms captured by $R^*(Q_{t-1}\tilde{K}_t - \tilde{N}_t)$, less total monitoring costs $\mu\int_0^{\bar{\omega}_t}\omega f(\omega)\,\mathrm{d}\omega\, R_t^K Q_{t-1}\tilde{K}$.[20]

To keep the number of entrepreneurs constant bankrupt firms are replaced in every period by "newborn" ones. In order to endow those starting entrepreneurs with some initial capital we assume that they also work and receive wages $\tilde{W}^e$. The net worth of the whole sector for the next period is then simply the ex-dividend value of the remaining fraction of firms, combined with the proceeds from their own work H^e:[21]

$$\tilde{N}_{t+1} = \phi\,\tilde{V}_t + \tilde{W}_t^e \tag{3.8}$$

It is important to realize that the zero-profit condition 3.4 can, after taking expectations, be interpreted as an economy-wide loan supply curve of the following form:

$$E_t\left\{\frac{R_{t+1}^K}{R^*}\right\} = E_t\left\{\left(\frac{1}{\Gamma\left(\bar{\omega}_{t+1}\right) - \mu G\left(\bar{\omega}_{t+1}\right)}\right)\left(\frac{\tilde{B}_{t+1}}{Q_t\tilde{K}_{t+1}}\right)\right\} \tag{3.9}$$

Clearly, it implies a positive relationship between the risk premium and leverage where, following BGG, $E_t R_{t+1}^K$ is a proxy for the domestic risky interest rate. In Figure 1 we have seen that both leverage and risk premium tend to have very similar dynamic patterns over the cycle and, in particular, they have a very similar degree of countercyclicality. We regard this as evidence that the majority of interest rate dynamics over the business cycle occurs *along* the loan supply curve and hence might be due to fluctuations in the demand for credit. This is because shocks to the demand for loans induce a positive comovement between the leverage and the premium, as Figure 1 shows. In fact, in the presence of TFP shocks only, the countercyclicality of the risk premium will always be exactly the same as the countercyclicality of leverage. More generally, a

[19]The maximization problem presented previously is algebraically equivalent to maximizing $E_{t-1}\tilde{V}_t$.

[20]Labor return and costs terms are supressed because of perfect competition in the labor market.

[21]Entrepreneurial labor is assumed to be inelastic and normalized to 1.

dominance of shocks that affect only the demand for credit guarantees that this pattern will be retained. On the other hand, any shocks to the financial accelerator, e.g., a risk shock in the spirit of Christiano et al. (2010) or a monitoring cost shock á la Levin et al. (2006) would affect the slope of the loan supply curve and possibly break this pattern. For these reasons we decided to abstain from shocks to the accelerator and work with a parsimonious model in which the TFP shock is the sole source of uncertainty.

3.2 Capital Producers

Entrepreneurs are not permanent owners of capital, which is used as an input for production. Instead, they purchase (or rent) the capital stock $\tilde{K}_t$ from perfectly competitive capital-producing firms at the end of period $t-1$ at price Q_{t-1}. This capital is used in production at t and its undepreciated part $(1-\delta)\tilde{K}_t$ is re-sold at price $\bar{Q}_t$ to capital producers once the production is over.[22] Capital producers combine this capital with new investment using the following technology:

$$\tilde{K}_{t+1} = (1-\delta)\tilde{K}_t + \tilde{I}_t - \frac{\varphi}{2}\left(\frac{\tilde{K}_{t+1}}{\tilde{K}_t} - g\right)^2 \tilde{K}_t \tag{3.10}$$

where the last term captures the presence of adjustment costs. The new capital stock $\tilde{K}_{t+1}$ is then re-sold at price Q_t to entrepreneurs and the cycle closes. Formally, capital producers solve the following profit-maximization problem:

$$\max_{\tilde{K}_{t+1},\tilde{I}_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[Q_t \tilde{K}_{t+1} - \bar{Q}_t (1-\delta)\tilde{K}_t - \tilde{I}_t\right]$$

subject to equation 3.10. We assume that capital-producing firms are owned by households (discussed below) and therefore use their subjective discounting factor β. The combined first order conditions give:[23]

$$Q_t = 1 + \varphi\left(\frac{\tilde{K}_{t+1}}{\tilde{K}_t} - g\right) + \beta E_t\left\{(1-\delta)Q_{t+1} - (1-\delta) - \varphi\left(\frac{\tilde{K}_{t+2}}{\tilde{K}_{t+1}} - g\right)\frac{\tilde{K}_{t+2}}{\tilde{K}_{t+1}} + \frac{\varphi}{2}\left(\frac{\tilde{K}_{t+2}}{\tilde{K}_{t+1}} - g\right)^2\right\} \tag{3.11}$$

From the point of view of capital producers the timing of events is as follows. At the dawn of t, the aggregate TFP shock becomes known. Because this determines the aggregate levels of $\tilde{Y}_t$ and R_t^K, all information necessary to determine $\tilde{I}_t$ and hence the supply of $\tilde{K}_{t+1}$ becomes known. This is when their maximization problem is solved. Therefore, the time t TFP shock affects both investment and the price of capital on impact.

[22] Depreciation of capital occurs not between $t-1$ and t, but during the production process.

[23] As discussed in Bernanke et al. (1999), the difference between Q_t and $\bar{Q}_t$ is of second-order importance and is therefore suppressed in further exposition.

3.3 Households

The small open economy is inhabited by a continuum of identical atomistic households. A representative household maximizes its expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(\tilde{C}_t - \tau \tilde{X}_t \frac{H_t^{\gamma}}{\gamma}\right)^{1-\sigma}}{1-\sigma}$$

where σ is the constant relative risk aversion coefficient (inverse of the intertemporal elasticity of substitution). Preferences are assumed to take the Greenwood et al. (1988) form. Households obtain income from working for the entrepreneurial sector. Their optimal labor supply function is given by

$$\tau \tilde{X}_t H_t^{\gamma-1} = \tilde{W}_t \tag{3.12}$$

This equation reflects the key property of GHH preferences, i.e., labor supply is not dependent on the level of consumption. In other words, the income effect on labor is absent. This in turn allows these preferences to replicate more closely some important business cycle properties for both developed (Correia et al. (1995)) and emerging (Neumeyer and Perri (2005)) economies.

In order to smooth consumption, households borrow funds from abroad. Although ex post consumers do not default, foreign lenders do not know this ex ante, e.g., because of information asymmetries, and hence charge a premium. In particular, the borrowing rate at $t-1$ is $E_{t-1}R_t^K$, which links it to the entrepreneurial sector. The budget constraint is given by

$$\tilde{C}_t - \tilde{D}_{t+1} = \tilde{W}_t H_t - \Psi_t E_{t-1} R_t^K \tilde{D}_t \tag{3.13}$$

where Ψ_t is the risk premium elasticity defined as

$$\Psi_t = \left\{ \bar{\Psi} + \tilde{\Psi} \left[\exp\left(\frac{\tilde{D}_t^A}{\tilde{X}_t} - d \right) - 1 \right] \right\} \tag{3.14}$$

where $\tilde{D}_t^A$ is the aggregate level of debt, equal to $\tilde{D}_t$ in equilibrium. The term $\bar{\Psi}$ allows us to calibrate the subjective discount factor β (see Appendix B for details). On the other hand, $\tilde{\Psi}$ is calibrated to a very low number and its sole purpose is to induce stationarity to net debt, consumption and the trade balance (see, e.g., Schmitt-Grohé and Uribe (2003)). It has no other bearing on the dynamics of the model. The optimal intertemporal consumption choice is given by the following Euler equation

$$\tilde{\lambda}_t = \beta E_t \Psi_{t+1} R_{t+1}^K \tilde{\lambda}_{t+1} \tag{3.15}$$

where $\tilde{\lambda}_t$ is the marginal utility of consumption defined as

$$\tilde{\lambda}_t = \left(\tilde{C}_t - \tau \tilde{X}_t \frac{H_t^{\gamma}}{\gamma} \right)^{-\sigma} \tag{3.16}$$

3.4 Labor Market and Remaining Specification

The aggregate production function is of the standard Cobb-Douglas type:

$$\tilde{Y}_t = A_t \tilde{K}_t^{\alpha} \left(\tilde{X}_t L_t \right)^{1-\alpha} \tag{3.17}$$

Recall that labor is supplied both by households and entrepreneurs. Therefore the total labor input L_t is the aggregate of the two:

$$L_t = \left(H_t^e \right)^{\Omega} H_t^{1-\Omega} \tag{3.18}$$

where the working hours of entrepreneurs H_t^e are normalized to 1 and Ω is the share of entrepreneurs' share of total labor. This gives rise to two separate labor demand functions:

$$(1-\alpha)\,\Omega \frac{\tilde{Y}_t}{H_t^e} = \tilde{W}_t^e \tag{3.19}$$

as well as

$$(1-\alpha)\,(1-\Omega)\,\frac{\tilde{Y}_t}{H_t} = \tilde{W}_t \tag{3.20}$$

We close the model by specifying the market clearing condition for final goods:

$$\tilde{Y}_t = \tilde{C}_t + \tilde{C}_t^e + \tilde{I}_t + \tilde{NX}_t + \mu \int_0^{\bar{\omega}_t} \omega f(\omega)\, \mathrm{d}\omega \, R_t^K Q_{t-1} \tilde{K}_t \tag{3.21}$$

The last term in this expression captures resources wasted for monitoring.

4 Parametrization and Estimation

We turn now to the empirical part of the exercise where we apply the model to emerging economies' data. In order to match the moments that distinguish these economies, as documented in Section 2, we estimate some of the key parameters of the model, including those of the financial contract. Since we want to focus on the role of the accelerator and do not want to attribute the results to idiosyncrasies in preferences, we calibrate this part of the model following the previous literature and the data. Table 6 summarizes the values that we use. We perform Generalized Method of Moments (GMM) estimation of two groups of parameters, i.e., those describing the financial accelerator mechanism, as well as the strength and persistence of the productivity shock. The parameters are listed in detail in Table 7. We choose the following nine second moments:

$$m(\boldsymbol{\theta}) = \left[\begin{array}{ccccccccc} \sigma^2(Y) & \sigma^2(C) & \sigma^2(I) & \sigma^2(TB) & \rho(TB,Y) & \rho(C,Y) & \rho(I,Y) & \sigma^2(R) & \rho(R,Y) \end{array} \right]' \tag{4.1}$$

where $\boldsymbol{\theta} = [\mu\ \sigma\ \varphi\ \phi\ \rho_A\ \sigma_A]'$ is the vector of parameters, σ^2 denotes a variance and ρ indicates a correlation coefficient. The moments' empirical counterparts are based on variables $\{Y_{i,t}, C_{i,t}, I_{i,t}, TB_{i,t}, R_{i,t}\}$, denoting output, consumption, investment, trade balance and domestic interest rate, respectively. Empirical moments

Table 6: Calibrated parameters.

Parameter	Description	Value	Source
g	deterministic trend (quarterly)	1.0090	data
$\frac{C}{Y}$	consumption to GDP ratio	0.724	data
α	capital share in production	0.32	Aguiar and Gopinath (2007a)
β	subjective discount factor	0.98	Aguiar and Gopinath (2007a)
γ	GHH labor parameter	1.6	Neumeyer and Perri (2005)
δ	depreciation rate	0.05	Aguiar and Gopinath (2007a)
σ	relative risk aversion	2	Aguiar and Gopinath (2007a)
Ω	entrepreneurial labor share	0.01	Bernanke et al. (1999)
R^*	foreign interest rate (annualized)	1.0077	data
H	steady state labor	0.33	Aguiar and Gopinath (2007a)

Table 7: Estimated parameters.

Parameter	Description
μ	monitoring costs
σ_ω	std dev. of idiosyncratic productivity
φ	capital adjustment costs parameter
	survival rate
ϕ	dividend parameter
ρ_A	persistence of TFP shock
σ_A	std dev. of TFP shock

were derived using HP cycle components of logs of series in levels. The exception is trade balance TB_t, i.e. the ratio of net exports to output $TB_t \equiv NX_t/Y_t$, where no logarithms were taken prior to HP filtering. The HP-filtered model moments were obtained using the procedure suggested by Burnside (1999). The dataset used in estimation is an unbalanced panel of the 13 emerging economies described in Section 2 between 1994 and 2010. Note that $m(\boldsymbol{\theta})$ does not include moments related to leverage. We exclude this variable for two reasons. First, since leverage series are shorter than others, we would be forced to reduce our sample by almost 200 observations including the 1997 crisis episode in South Korea. Secondly, as discussed in Section 3.1, a model with TFP shocks only will always predict the same degree of cyclicality for both the risk premium and leverage. Therefore, by targeting interest rate cyclicality we automatically target leverage cyclicality as well, a very close number, as we know from Section 2.

In order to account for possible autocorrelation as well as cross-correlation across countries, we apply the Driscoll and Kraay (1998) estimator, a modification of the HAC estimator adjusted for panel data which allows for cross-correlations of errors. The estimator involves a minimization of expected (over time) values of cross-country average errors:

$$\frac{1}{T}\sum_{t=1}^{T}\boldsymbol{h}_t\left(\boldsymbol{\theta}\right) \qquad \text{with} \qquad \boldsymbol{h}_t\left(\boldsymbol{\theta}\right)=\frac{1}{N(t)}\sum_{i=1}^{N(t)}\boldsymbol{h}_{i,t}\left(\boldsymbol{\theta}\right)$$

where $\boldsymbol{h}_{i,t}\left(\boldsymbol{\theta}\right)$ is a 9×1 vector of the following moment errors:

$$\boldsymbol{h}_{i,t}\left(\boldsymbol{\theta}\right)=\left\{\begin{array}{c} m_1\left(\boldsymbol{\theta}\right)-Y_{i,t}^2 \\ \frac{m_2(\boldsymbol{\theta})}{m_1(\boldsymbol{\theta})}-\frac{C_{i,t}^2}{m_1(\boldsymbol{\theta})} \\ \frac{m_3(\boldsymbol{\theta})}{m_1(\boldsymbol{\theta})}-\frac{I_{i,t}^2}{m_1(\boldsymbol{\theta})} \\ \frac{m_4(\boldsymbol{\theta})}{m_1(\boldsymbol{\theta})}-\frac{TB_{i,t}^2}{m_1(\boldsymbol{\theta})} \\ m_5\left(\boldsymbol{\theta}\right)-\frac{TB_{i,t}Y_{i,t}}{\sqrt{m_1(\boldsymbol{\theta})m_4(\boldsymbol{\theta})}} \\ m_6\left(\boldsymbol{\theta}\right)-\frac{C_{i,t}Y_{i,t}}{\sqrt{m_1(\boldsymbol{\theta})m_2(\boldsymbol{\theta})}} \\ m_7\left(\boldsymbol{\theta}\right)-\frac{I_{i,t}Y_{i,t}}{\sqrt{m_1(\boldsymbol{\theta})m_3(\boldsymbol{\theta})}} \\ \frac{m_8(\boldsymbol{\theta})}{m_1(\boldsymbol{\theta})}-\frac{R_{i,t}^2}{m_1(\boldsymbol{\theta})} \\ m_9\left(\boldsymbol{\theta}\right)-\frac{R_{i,t}Y_{i,t}}{\sqrt{m_1(\boldsymbol{\theta})m_8(\boldsymbol{\theta})}} \end{array}\right\} \tag{4.2}$$

We start the estimation with an identity matrix and re-estimate the model using the optimal weighting matrix obtained in the first step. The estimator of the GMM weighting matrix $\boldsymbol{W}$ is given by

$$\hat{\boldsymbol{W}}^{-1}=\hat{\boldsymbol{\Omega}}_0+\sum_{j=1}^{m(T)}\left[1-\frac{j}{m\left(T\right)-1}\right]\left(\hat{\boldsymbol{\Omega}}_j+\hat{\boldsymbol{\Omega}}_j'\right)$$

with

$$\hat{\boldsymbol{\Omega}}_0=\frac{1}{T}\sum_{t=1}^{T}\boldsymbol{h}_t(\hat{\boldsymbol{\theta}})\boldsymbol{h}_t(\hat{\boldsymbol{\theta}})' \qquad \text{and} \qquad \hat{\boldsymbol{\Omega}}_j=\frac{1}{T}\sum_{t=j+1}^{T}\boldsymbol{h}_t(\hat{\boldsymbol{\theta}})\boldsymbol{h}_{t-j}(\hat{\boldsymbol{\theta}})' \qquad \text{where} \qquad \boldsymbol{h}_t(\hat{\boldsymbol{\theta}})=\frac{1}{N(t)}\sum_{i=1}^{N(t)}\boldsymbol{h}_{i,t}(\hat{\boldsymbol{\theta}})$$

and m is the number of lags. We follow a commonly used criterion $m(T)=\left\|0.75T^{1/3}-1\right\|$, which yields $m=2$ in our panel. Note also that $N(t)$ varies over time.

5 Results

This section presents the main results of the GMM estimation. We assess the model performance in terms of matching the key moments for emerging economies as well as the dynamics of leverage. We also report the estimated parameters and document their similarities and differences with other studies. However, a further exploration of the link between the parameters and the model's performance is postponed until the next section.

5.1 Main Business Cycle Moments

Table 8 presents the model's performance along the moments presented in table 1 while table 9 reports the estimated parameter values. The most important result that emanates from table 8 is that the model is able to reproduce the dynamics of interest rates for emerging economies, i.e. their volatility and countercyclicality. Simultaneously, the model performs well in terms of matching the other seven moments that characterize the data. In particular, it is able to generate a high volatility of output, despite slightly overestimating it, as well as the relative volatility of investment. As in the data, consumption in the model is more volatile than output. Also, the model is able to reproduce the behavior of the trade balance, both in terms of its volatility and countercyclicality.

Table 8: Model generated moments for emerging markets.

Second Moment	**Emerging Markets**	**Model**
$\sigma(Y)^a$	3.62 (0.27)[b]	4.03 (0.16)
$\sigma(C)/\sigma(Y)$	1.09 (0.07)	1.11 (0.04)
$\sigma(I)/\sigma(Y)$	3.41 (0.28)	3.32 (0.15)
$\sigma(TB)$	3.06 (0.30)	3.27 (0.24)
$\rho(TB,Y)$	−0.30 (0.07)	−0.33 (0.04)
$\rho(C,Y)$	0.73 (0.05)	0.95 (0.01)
$\rho(I,Y)$	0.71 (0.04)	0.73 (0.02)
$\sigma(R)$	3.57 (0.21)	3.30 (0.30)
$\rho(R,Y)$	−0.35 (0.06)	−0.35 (0.03)

[a] Standard deviations are expressed in %.

[b] Standard errors are reported in brackets.

Table 9: Estimated parameter values.

Parameter	μ	σ	φ	ϕ	ρ_A	σ_A
Estimated value	0.384	0.138	9.931	0.678	0.999	0.016
	(0.701)	(0.025)	(0.636)	(0.273)	(0.003)	(0.001)

The procyclicality of investment in the model is also in line with the data. The model performs slightly worse in terms of the comovement of consumption with output. In the model consumption correlation is as high as 0.95, as opposed to 0.73 in the data. Although the model doesn't perform well in this dimension, it is also true that the empirical moment that we try to match differs from what has been reported in previous studies[24].

These results illustrate that a model in which interest rate dynamics are endogenously driven by variation in the risk premium markup in the financial accelerator serves well in accounting for some of the main business cycle patters in emerging economies. The results are a function of the values of the parameters estimated in the GMM, particularly those that define the financial contract. These values are reported in Table 9.

Arguably, the most remarkable result in Table 9 is the value taken by ϕ, equal to 0.678. It is significantly lower than what has been commonly used in previous studies using the BGG framework. For quarterly frequency (and for developed economies), it has usually been set in the range of $0.9728 - 0.99$, although Carlstrom and Fuerst (1996) have set it to 0.878[25].

The origins of ϕ are purely technical in the BGG framework. In particular, in a model where ϕ converges to 1 entrepreneurs would be able to accumulate capital until they became totally self-financed and so the agency problem would disappear. Traditionally, $1 - \phi$ has been interpreted as the fraction of firms that leave the market despite not having defaulted in a given period. For BGG calibration $\phi = 0.9728$ this would translate into almost 37 quarters, or over nine years of firms' average lifetimes. Finance literature on deaths and life cycles of firms estimates average life expectancies of, roughly, 7-11 years[26] based on firm registers in the United States.[27] However, the predominant reason why firms disappear from registers is *exactly* bankruptcy.[28] Therefore, following this interpretation, ϕ may be significantly underestimated. Nevertheless, this parameter can be interpreted in at least a couple of alternative ways. Given that there is a continuum of atomistic firms in the model, one may think of them as individual production lines rather than actual firms. Alternatively, $1 - \phi$ may reflect firms that have abandoned the credit market, possibly because they have become self-financed. These would render the high values of ϕ empirically more plausible. However, given that a fraction $1 - \phi$ of the net profit of firms V_t in the model is passed for (entrepreneurial) consumption C_t^e, the most natural interpretation for this parameter is that of a dividend paid to shareholders, as presented in

[24] For example, Aguiar and Gopinath (2007a) match only the correlation of Mexico, which they report to be 0.92. Their model also generates correlations above 0.9, depending on the specification. In Neumeyer and Perri (2005) the reported empirical correlation for emerging economies is around 0.8. Yet, they match the correlation of Argentina, 0.97. Depending on the version, their model generates correlations between 0.82 and 0.97. Also, it is worth noting that our reported empirical moments include only private consumption and, for consistency, output net of public consumption.

[25] We cite the working paper version of the Carlstrom Fuerst study because, similarly to our model, it includes the parameter ϕ, making it more comparable to our work than the published Carlstrom and Fuerst (1997) version, which lacks this feature.

[26] See Morris (2009) for an informative survey.

[27] Standard sources are the U.S. Small Business Database of the Small Business Administration, the Thomas Register of American Manufacturers and the U.S. Census of Manufacturers.

[28] Other popular reasons include mergers, acquisitions and takeovers.

Section 3. In particular, it corresponds to the dividend payout ratio used in corporate finance.[29] A somewhat similar interpretation has been used by Gertler and Kiyotaki (2010), where ϕ occurs in the context of banks' equity. Indeed, empirical evidence for this financial measure is broadly in line with our estimated value of ϕ. Table 10 reports average dividend payout ratios for our sample of emerging economies.[30] Clearly, our

Table 10: Average dividend payout ratios across emerging economies.

Argentina	25.90%	Korea	17.20%
Brazil	38.36%	Malaysia	28.97%
Chile	26.29%	Philippines	18.78%
Colombia	61.53%	South Africa	27.14%
Mexico	15.55%	Thailand	45.93%
Peru	31.82%	Turkey	27.23%
Average	**30.39%**		

estimated value of $1 - \phi = 0.322$ is very close to the average dividend payout ratio found in the data, 0.304. Although a thorough analysis of dividend behavior in emerging economies is clearly beyond the scope of this paper, this evidence clearly speaks in favor of our estimated ϕ value and its interpretation as dividend payout ratio.

Why does the GMM estimation favor low values of ϕ? While a careful exploration of that question is provided in the following section, we point out here that this parameter reflects the "leverage mechanism" at work in our model. The parameter plays a key role in determining the high steady-state levels of leverage and risk premium and, ultimately, the model's performance, particularly in terms of the dynamics of interest rates and leverage. The leverage level implied by our estimated value of ϕ is $\frac{QK}{N} = 6.345$, whereas the risk premium is $\frac{R^K}{R^*} = 1.088$. Both values are determined by the steady state default productivity cutoff $\bar{\omega} = 0.791$, which implies a default rate in the optimal contract of 5.2 percent, or 19.37 percent annualized. This is a significantly higher number than those seen in some previous studies, e.g., 3 percent annualized in BGG. The data on failure rates beyond the United States is scarce and also poses considerable problems of interpretation. The only multi-country study which reports official bankruptcy rates that we are aware of is that of Claessens and Klapper (2005). According to their data, the average annual rate for Argentina, Chile, Colombia, Peru, Korea and Thailand is 0.15 percent a year, as opposed to, e.g., 4.62 percent for South Africa. Therefore, the official rates seem to reflect much more the legal system of a country rather than pure

[29]The analogy is not perfect because of the existence of the W_t^e term, which is nevertheless of negligible size in our model.

[30]The data is taken from Bloomberg. See appendix A for details.

economics and are therefore not directly comparable in economic terms.[31] Lastly, the elasticity of 0.108 is a bit larger than in other studies that work with developed countries (in the range of $0.04-0.08$).

The estimated monitoring cost fraction μ of 0.384 is larger, albeit with a high degree of uncertainty,[32] than the value 0.12 calibrated originally by BGG based on U.S. data. It is in the upper range of other studies focusing on the United States and Europe. For example, Carlstrom and Fuerst (1997) consider calibrations for the U.S. with 0.2, 0.25 and 0.36. Using Bayesian techniques, Queijo von Heideken (2009) reports posterior means of 0.159 for the U.S. and 0.271 for the Euro Area. Using a partial equilibrium model, Levin et al. (2006) show how these costs have varied over time in the U.S. case. In their estimation, they ranged between $[0.1, 0.3]$ over the 1997-2000 period, but then oscillated between 0.3 and 0.5 in the years 2000-2003. In the study of Fuentes-Albero (2012) the U.S. number was 0.24 until 1983, but only 0.04 from 1984 on. However, that study reports a major increase in the volatility of monitoring costs during the Great Moderation era — a parallel "financial immoderation". On the other hand, Christiano et al. (2010) calibrate this parameter to as much as 0.94 in order to match other steady state values. A proxy of direct costs can be also found in the Doing Business database of the World Bank. The average cost of closing a business (expressed as a percent of estate) is 16.08 percent for our sample of 13 developing and 6.46 percent for the sample of small open developed economies. Yet, as argued by Carlstrom and Fuerst (1997), such costs are only direct and they do not include other indirect costs. We share their view that μ should be regarded in this broader sense. The relatively high value of monitoring costs should be treated as a broad indicator that financial frictions are at work in emerging market economies, possibly even more so than in developed ones.

The value of σ, the standard deviation of the idiosyncratic productivity, is estimated to be 0.138, a number similar to those used in the literature. The numbers reported for the U.S. range from 0.088 in Carlstrom and Fuerst (1996) and 0.15 in Queijo von Heideken (2009) to 0.529 in the original BGG paper. For the Euro Area, the latter study reports 0.21, similar to 0.24 in Christiano et al. (2010).

The GMM estimation points to the capital adjustment costs parameter value of 9.931. This is a reduced-form parameter and its value depends on the functional specification of capital adjustment costs. Since there is no consensus on its feasible value range, it suffices to say that our estimate is broadly in line with previous literature. In particular, our estimated value is higher than $\varphi = 4$ calibrated in Aguiar and Gopinath (2007a) and $\varphi = 4.6$ estimated in García-Cicco et al. (2010). Yet, it is similar or significantly lower than the values used in Neumeyer and Perri (2005), which range between 8 and 40, or $\varphi = 72.8$ estimated in Uribe and Yue (2006).

[31] As another example, compare the official annual bankruptcy rate for Spain which is 0.02 percent versus 3.65 percent for the United States or 2.62 percent for France.

[32] In the next section we analyze more extensively the sources of the high uncertainty around the point estimate of μ.

While the TFP shock volatility of 1.6 percent is a number similar to the values reported in previous studies for emerging economies,[33] the autoregressive component, $\rho_A = 0.999$, essentially points to unit root persistence of the productivity shock. This is in contrast with the value of $\rho_A = 0.95$ commonly found in other studies that have used calibration or GMM methods. Thus, our estimate clearly suggests a significant role for a "trend shock" as in Aguiar and Gopinath (2007a). This result is also important in light of the way in which we model the consumers' sector in section 3.3. In our specification, households have to pay $E_{t-1}R_t^K$ on their debt, a risky rate linked directly to the entrepreneurial sector and to the financial accelerator. They have to do so despite the fact that households do not explicitly default in the model. This modeling device is somewhat analogous to reduced-form financial frictions found in many previous studies (e.g., in Neumeyer and Perri (2005), Uribe and Yue (2006) or García-Cicco et al. (2010)) where risk premia are built into consumers' budget constraints. Despite having a similar mechanism in our model, the GMM estimation still points to unit root persistence in the total factor productivity process. In fact, in some further robustness checks (results upon request) we found that the dynamics of the model is not much affected when $E_{t-1}R_t^K$ is replaced with a riskless international interest rate R^*. We therefore interpret it as evidence that "the cycle is the trend" hypothesis cannot be dismissed even when one explicitly considers financial frictions in the form of a financial accelerator. In subsection 6.3 we look closer at the role of TFP persistence in driving the results.

5.2 Leverage Dynamics

A natural next step is to ask to what extent can the estimated model replicate the leverage patterns depicted in Figure 1. Our model proxy for the empirical assets-to-equity ratio analyzed in Section 2 is the expression $(Q_t K_{t+1})/N_{t+1} = (N_{t+1} + B_{t+1})/N_{t+1}$, where firms' assets are represented by $Q_t K_{t+1}$, debt by B_{t+1} and equity by N_{t+1}. Note that there is no secondary market for debt in the model. This makes the variable B_{t+1} somewhat similar to the book value of debt observed in the data. On the other hand, capital in the model is traded in every period at market price Q_t. Therefore N_{t+1} constitutes a good model counterpart of the empirical market value of equity measured by total market capitalization. Table 11 reports the model-generated serial correlations between leverage and output together with their empirical counterparts from Figure 1[34]. What can be seen is that, qualitatively, the model is able to reproduce a considerable part of data dynamics. A good fit of the instantaneous correlation follows from the fact that the model-generated cyclicality of interest rates is the same as the cyclicality of interest rates by construction. Since these two values are similar in our data and the model does a good job of matching interest rate cyclicality, a good

[33] For example, Neumeyer and Perri (2005) set it between 1.47% and 1.98%, depending on the version of the model.

[34] The empirical numbers in the table differ very slightly from those in figure 1. The numbers in the table are obtained by GMM estimation on an unbalanced panel, whereas in the figure the leverage is a simple average across countries.

Table 11: Model generated leverage dynamics $Corr(Y_t, Lev_{t+j})$. Standard errors reported in brackets.

j	-4	-3	-2	-1	0	1	2	3	4
Model	-0.261	-0.311	-0.355	-0.379	-0.350	0.265	0.443	0.451	0.396
	(0.019)	(0.022)	(0.023)	(0.023)	(0.029)	(0.043)	(0.01)	(0.020)	(0.032)
Data	-0.016	-0.163	-0.293	-0.343	-0.351	-0.206	-0.075	0.041	0.127
	(0.053)	(0.050)	(0.075)	(0.087)	(0.102)	(0.070)	(0.067)	(0.045)	(0.050)

match for leverage follows. In addition to this, the model captures many leads and lags correlations. In particular, it is able to roughly replicate the countercyclicality of leverage lags with the cycle (in the data the correlations are slightly lower than in the model). It also replicates the procyclicality of leverage leads, although it overstates it. If a recession hits at $j = 0$, the deleveraging in the model occurs much more abruptly than in the data, where it is more moderated and prolonged. We consider this to be a satisfactory result given that lags and leads of leverage were not even a part of the GMM objective function. It should be noted, however, that the model performs less satisfactory in terms of leverage volatility, the standard deviation of which is 13.65 percent in the data. Given the model-implied risk premium elasticity, in order for the model to match the empirical interest rate volatility, the standard deviation of leverage needs to be as large as 30.68 percent.

Summing up, the results reported in this section show that the estimated model can successfully account for many of the documented business cycle patterns in emerging economies, in particular the dynamics of interest rates and leverage. The model's relatively good performance is linked to the presence of an endogenous risk premium markup stemming from a financial accelerator. We thereby offer a structural, yet tractable, mechanism by which some of these business cycle patterns can be rationalized. Finally, we show that these results were obtained by estimating some structural parameters in the financial contract at values different than those commonly used in calibrations. In particular the estimation chooses a value of ϕ that is in line with dividend payout ratios observed in emerging economies. Our results also indicate that emerging economies data can be seen through the lens of a model characterized by a relatively high level of *steady state* leverage. In the next section we further explore this issue.

6 Inspecting the Mechanism

In what follows, we inspect the mechanism by which the results of the GMM estimation reported in the previous section are obtained. In particular, we focus on the role played by the estimated parameters in determining the model's performance when accounting for interest rate dynamics in emerging economies, namely their high volatility and countercyclicality. To do so we start by analyzing the impact of the estimated parameter values on the non-stochastic steady state. We show that some of the parameters estimated in the financial contract, most notably ϕ, define the steady-state leverage and this in turn defines the dynamics of interest rates which we map to the data. We illustrate this by documenting how impulse response functions, following a TFP shock, vary substantially across different steady states of leverage. Finally, we perform a series of simulations with counterfactual parameterizations.

6.1 Steady State

We start by providing some intuition behind the impact of different parameter values on the non-stochastic steady state. We are particularly interested in studying the impact of the parameters in the financial contract on the steady state levels of leverage, risk premium, default rate and the elasticity of the risk premium to the leverage ratio. To do so we start from the equation that determines the optimal steady state cutoff $\bar{\omega}$:[35]

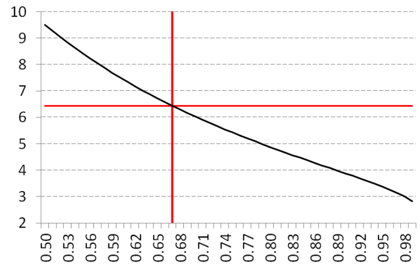
$$s(\bar{\omega}) - \frac{1-\delta}{R^*} = \frac{\alpha}{\Omega(1-\alpha)} \left[\frac{g}{R^*} \frac{1}{k(\bar{\omega})} - \phi(1 - \Gamma(\bar{\omega})) s(\bar{\omega}) \right] \tag{6.1}$$

where $s(\bar{\omega}) = \frac{R^K}{R^*}$ and $k(\bar{\omega}) = \frac{QK}{N}$ are the risk premium and leverage, respectively. This equation can be treated as an implicit function of optimal solvency threshold $\bar{\omega}$ conditioned on the levels of the other parameters, most notably the estimated parameters in the financial contract: $\{\mu, \sigma, \phi\}$. Furthermore, it can be shown that the default rate, $F(\bar{\omega})$, and the elasticity of the risk premium to the leverage ratio, $\eta_{s,k}$, are both functions of $\bar{\omega}$, and thus are also influenced by these parameters.

We perform the following three comparative statics experiments. First, we assess how the steady state is affected by different values of ϕ while fixing the other parameters according to the estimation results reported in Section 5.[36] This experiment is summed up in Figure 2, where the red crosses denote the estimated parameter value. The most remarkable result of this experiment is that, as we move to higher levels of ϕ, the steady-state level of leverage falls significantly, dropping below 3 for ϕ close to 1, as seen in 2(a). This pattern can be intuitively explained with eq. 3.8 which is used to derive 6.1. The higher the ϕ, the higher, *ceteris paribus*, is the net worth and hence lower the leverage. This also implies lower steady-state levels of risk premium (subfigure 2(b)). As the economy gets less leveraged, the markup over the risk-free interest rate almost disappears and the economy-wide risk gets lower, too. Entrepreneurial default rates follow a similar pattern, i.e., they re high for high dividend rates $1-\phi$ and relatively low for low ones. Lastly, the elasticity of the risk premium with respect to leverage exhibits a hump shape. It goes up as we move to higher dividend rates, but then starts falling as ϕ reaches values below, roughly, 0.85. In the second experiment, reported in

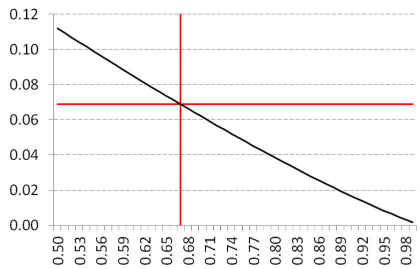
[35] See Appendix B for a detailed derivation.

[36] Although this can in principle be done by obtaining a closed-form solution to the implicit derivatives, the algebra becomes extremely elaborate and therefore we proceed with numerical simulations.



(a) Leverage

(b) Risk premium



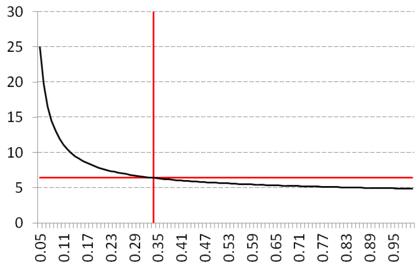
(c) Default rate

(d) Elasticity

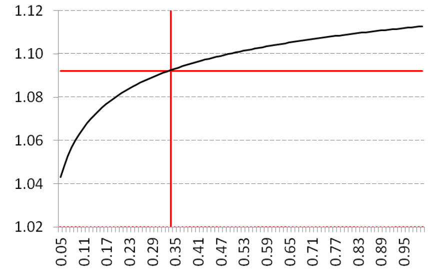
Figure 2: Steady state characteristics under different ϕ.

figure 3, we manipulate the monitoring costs μ. As they get lower, the economy approaches a model with no asymmetric information. In consequence, the risk premium approaches zero and optimal leverage becomes unbounded. Similarly, the risk premium elasticity fades away to zero. Note also that the curves around the estimated value are relatively flat. This explains the high standard error of the estimated μ reported in the previous subsection. This parameter is much better identified at lower-value intervals. Finally, we vary the standard deviation of idiosyncratic productivity σ, which is summed up in Figure 4. This parameter has an impact on the steady state mainly because of the asymmetry of the log-normal distribution function. To some extent, the impact of varying sigma is similar to that of μ. In particular, steady-state leverage is higher for low idiosyncratic productivity volatility. Risk premium rises as volatility goes up, as does the default rate. Finally, the elasticity of the premium dies out as σ falls.

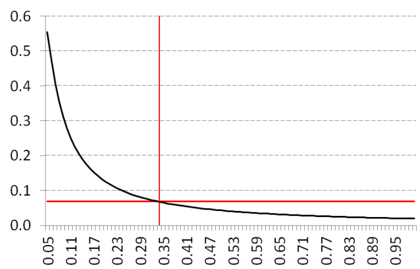
One can explain these results by analyzing the steady-state position of the supply and demand curves on the credit market. Changing μ as well as σ translates into a change in the costs of borrowing. This in turn affects the position of the loan supply curve 3.9 while keeping the demand curve fixed. That shift induces a negative comovement between capital and leverage on the one hand and the risk premium on the other. This can be seen by confronting the subfigures for leverage (a decreasing function of μ and σ) and the risk premium (an increasing function of μ and σ). Varying the dividend rate parameter ϕ, on the other hand, moves the demand for loans, while keeping the loan supply curve 3.9 unchanged, as shown in Figure 2. This induces a positive comovement between leverage and the risk premium.



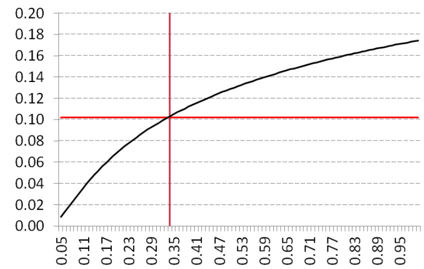
(a) Leverage



(b) Risk premium



(c) Default rate



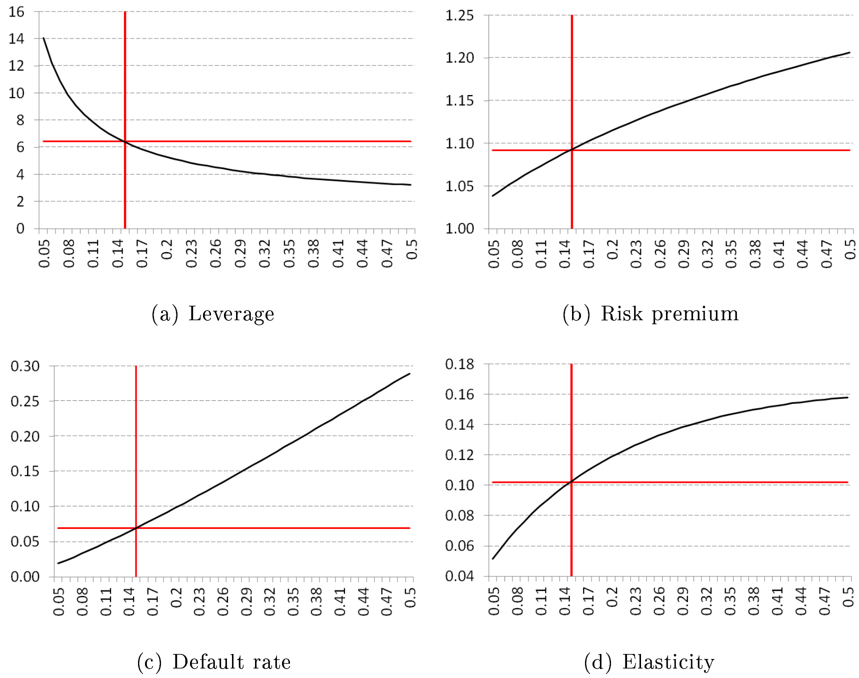
(d) Elasticity

Figure 3: Steady state characteristics under different μ.

6.2 Dynamics and Impulse Responses

We now move to the analysis of the model dynamics by assessing the impulse response functions across various parameterizations of the steady state. The results are reported in Figures 5(a) through 8. However, before we look at the results in detail, we believe some economic intuition is needed.

Recall that the risky domestic interest rate is proxied by $E_t R^K_{t+1}$ and that the economy is hit solely by stationary total factor productivity shocks. Therefore, in order to match the data for emerging economies, the best estimation has to point to parameter values for which, following a positive shock, GDP rises on impact, and the interest rate falls rather strongly. After a positive TFP shock in period t the marginal productivity of capital R^K_t goes up, thus increasing the value of the firm (see equations 3.3 and 3.7) and increases entrepreneurial net worth (eq. 3.8). This per se does not yet determine the behavior of the future expected return $E_t R^K_{t+1}$. Whether the interest rate will actually fall, and by how much, depends on the change of the net worth N_{t+1} relative to the change in total assets $Q_t K_{t+1}$. If the net worth goes up by relatively little (relative to assets), leverage will go up and, according to the loan supply curve (eq. 3.9), so will the premium over the riskless rate. However, this would translate into risk premium and interest rate procyclicality, which is counterfactual. Therefore, for the premium to fall, the net worth has to go up by more than assets, i.e., leverage has to fall on impact. Additionally, this drop in leverage has to be quite substantial in order to generate a sufficient response of interest rates.



(a) Leverage

(b) Risk premium

(c) Default rate

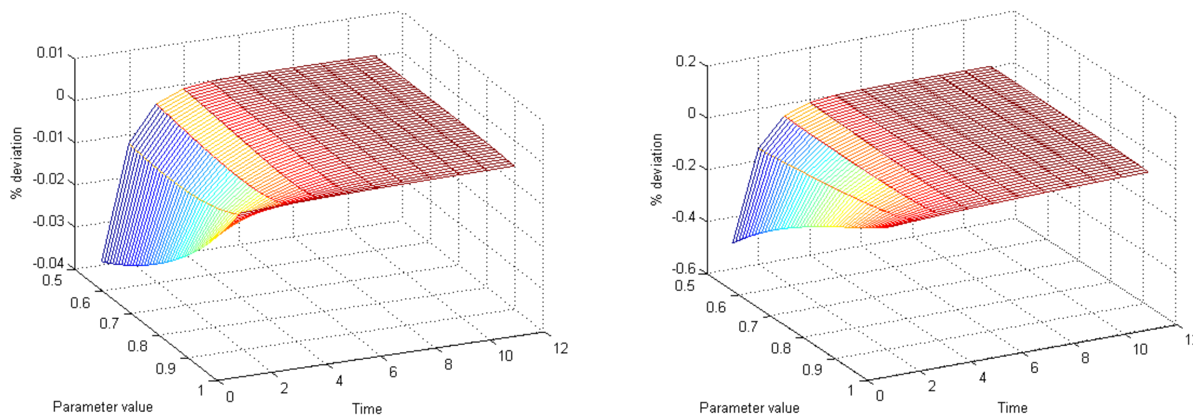
(d) Elasticity

Figure 4: Steady state characteristics under different σ.

The way to achieve a large increase in the entrepreneurial firm value V_t and, in consequence, in N_{t+1} after a positive shock is to be highly leveraged in the steady state. The same positive shock would generate smaller profits for a less leveraged economy than for a more leveraged one.[37] As discussed previously, it is precisely the low value of ϕ that allows for a high leverage of the economy in the non-stochastic steady state. This can be seen by inspecting Figures 5 through 8 where we report the impulse responses of the key variables over 12 quarters following a one-standard deviation positive shock to TFP. The figures are plotted in three dimensions, as we also report the sensitivity of such impulses to different levels of ϕ while all the other parameters are set at their estimated values. As the dividend parameter ϕ decreases (higher steady state leverage), both capital and its price increase. However, the net worth increases by even more. In consequence, leverage starts falling more abruptly on impact. In sum, the initial steady state leverage is high, but after a shock it falls significantly due to a windfall of profits. This in turn drives the risk premium

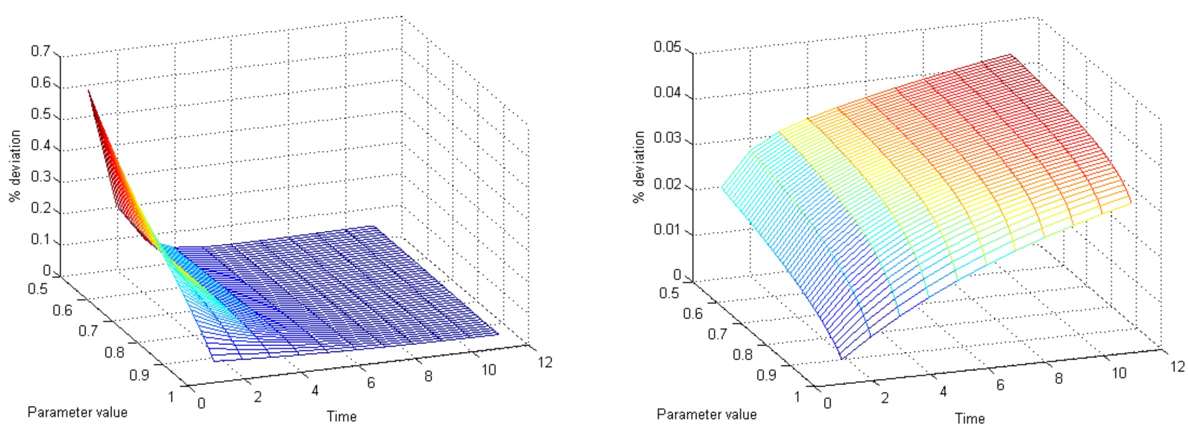
[37]A simple example helps to see this. Consider an economy with a return on investment $R^K = 5\%$ and borrowing cost $R^* = 1\%$. There are two firms in these economy. Firm X is highly leveraged. It borrows $B = 900$ and has net worth $N = 100$, so that $K = 1,000$. Firm X's revenue is $1,000 \times 1.05 = 1,050$ and debt payments are $900 \times 1.01 = 909$. Net income is $1,050 - 909 = 141$ and this is the new net worth of the firm. The net worth increase is therefore 41%. Firm Y is lowly leveraged. It borrows $B = 100$ and has equity $N = 900$, so again $K = 1,000$. Firm Y's revenue is $1,000 \times 1.05 = 1,050$ and debt payments $100 \times 1.01 = 101$. Net income (and, in consequence, the new net worth) is $1,050 - 101 = 949$. The net worth increase is approximately 5%.

and the interest rate down. In terms of the credit market demand and supply curves, that adjustment can be depicted as an inward shift of the loan demand curve. Compare this to a situation with high ϕ, e.g.



(a) Domestic interest rate $E_t R^K_{t+1}$ (b) leverage $\frac{Q_t K_{t+1}}{N_{t+1}}$

Figure 5: Responses of the domestic interest rate and leverage after a TFP shock for different values of ϕ.

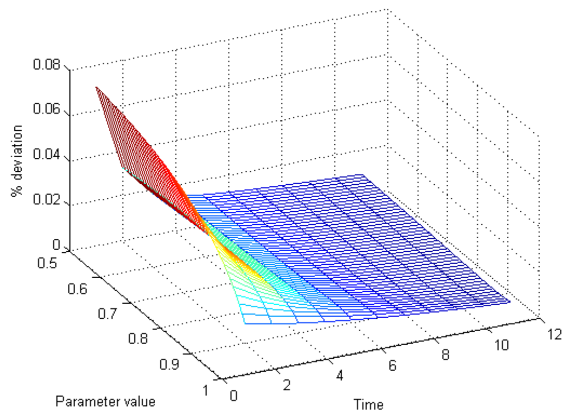


(a) Net worth N_{t+1} (b) Borrowing B_{t+1}

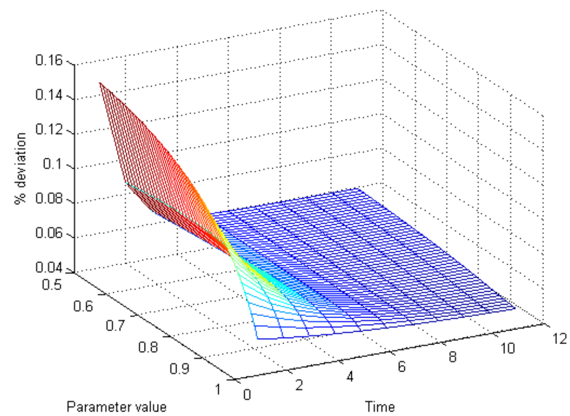
Figure 6: Responses of net worth and borrowing after a TFP shock for different values of ϕ.

$0.97-0.99$, as used in the literature for developed economies. The dynamics are now very different. Since the corresponding steady state leverage is relatively very low, entrepreneurial profit is reduced and the increases in V_t and N_{t+1} become low as well. With capital adjustment costs unchanged, assets $Q_t K_{t+1}$ increase on impact by only slightly less than net worth. Also, all these variables respond by much less in absolute terms. In consequence, both leverage and the interest rate go down on impact by only very little, which can be seen in Figures 5(a) and 5(b). In fact, if capital adjustment costs were slightly lower, the response of $Q_t K_{t+1}$

would become larger than that of N_{t+1} and in consequence both leverage and interest rates would become procyclical, as is the case for a standard BGG parameterization. Most importantly, in this case the strong volatility of leverage and, in consequence, of interest rates vanishes. To fully understand the model



(a) Price of capital Q_t



(b) Investment I_t

Figure 7: Responses of the price of capital and investment after a TFP shock for different values of ϕ.

dynamics, consider the market for capital. A significant increase in the net worth allows for a major rise in assets and hence generates a very high demand for capital. Since capital is predetermined on impact, this demand is reflected in a large increase in capital price Q_t as well as investment I_t, as can be seen in Figure 7. Also, although this increase in assets comes predominantly from new equity (internal funding), borrowing goes up slightly as well, as can be seen in Figure 6(b). This is because lower leverage has lowered external funding costs. In the period after the shock (i.e., at $t+1$) the price of capital falls significantly. First, the

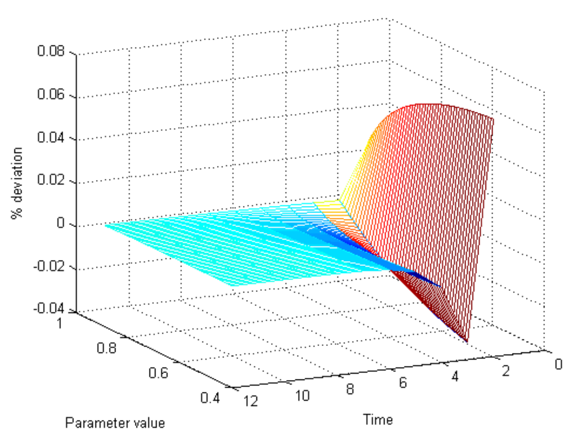


Figure 8: Responses of the return on capital R_t^K after a TFP shock for different values of ϕ (inverted picture).

supply of capital is now higher due to high investment at t. Secondly, the demand is now lower. This is due to the fact that leverage has fallen in the previous period t (on impact) and limited the increase in V_{t+1} and N_{t+2} relative to the previous period. In consequence, there iss a capital loss between t and $t+1$ and the return on capital in $t+1$ falls, as depicted in Figure 8 (note that the graph is turned around). Since this mechanism is expected as of t, it further decreases $E_t R^K_{t+1}$ and allows the model to match the large interest rate volatility in emerging economies.

6.3 Counterfactual Experiments

Another way to inspect the mechanism behind the results of the GMM estimation is to conduct counterfactual experiments. This is what we do in this last subsection. In particular, we analyze changes in the key business cycle moments derived from our model, relative to the benchmark estimation, when one parameter is manually set at a different value. We counterfactually set the values of μ, σ and ϕ to, roughly, those used in BGG and papers that followed this framework in the literature on developed economies. Table 12 summarizes these experiments. The first two columns reproduce the empirical moments and those derived from the GMM estimation. Columns 3 to 5 report the results when we counterfactually modify ϕ, μ, and σ. Column 6 presents the results when we manually modify the persistence of the stationary productivity shock, ρ_A. The most important message from the third column is that with high ϕ (i.e. low dividend rates) the volatilities

Table 12: Model generated moments with counterfactual parameters.

Moment	**Data**	**Model**	$\boldsymbol{\phi = 0.98}$	$\boldsymbol{\mu = 0.12}$	$\boldsymbol{\sigma = 0.53}$	$\boldsymbol{\rho_A = 0.95}$
$\sigma(Y)$ [a]	3.62	4.03	3.75	4.02	3.85	3.93
$\sigma(C)/\sigma(Y)$	1.09	1.11	1.13	1.14	1.11	0.82
$\sigma(I)/\sigma(Y)$	3.41	3.32	2.14	3.15	2.63	2.78
$\sigma(TB)$	3.06	3.27	1.99	3.28	1.42	2.39
$\rho(TB,Y)$	−0.30	−0.33	−0.72	−0.45	−0.44	−0.06
$\rho(C,Y)$	0.73	0.95	0.99	0.96	0.98	0.96
$\rho(I,Y)$	0.71	0.73	0.93	0.79	0.88	0.75
$\sigma(R)$	3.57	3.30	0.39	2.06	1.20	3.01
$\rho(R,Y)$	−0.35	−0.35	−0.69	−0.45	−0.48	−0.49
$\sigma(Lev)$	14.04	30.68	5.63	50.09	6.07	27.92
$\rho(Lev,Y)$	−0.35	−0.35	−0.69	−0.45	−0.48	−0.49

[a] Standard deviations are expressed in %.

of leverage and interest rate drop by an order of magnitude. Essentially this column captures the low steady state leverage that is driven by high ϕ, as documented in the previous subsection (see figure 5(a)). The fact that the interest rate is still strongly countercyclical stems from the fact that capital adjustment costs are too high for this parameterization. It lowers the volatility of investment (a drop to 2.14%), but also reduces the response of Q_t and K_{t+1} relative to N_{t+1}. This in turn generates a countercyclical leverage and, in consequence, interest rates. Setting $\phi = 0.98$ and, simultaneously, e.g. $\varphi = 1$ would increase the relative volatility of investment to 3.71 but also generate procyclical leverage and interest rates. This is because capital price Q_t and quantity K_{t+1} would become more volatile than net worth N_{t+1}[38]. The dynamics of the trade balance are affected by the change in investment dynamics. However, consumption remains largely unaffected[39].

Turning to column four, a lower μ increases steady-state leverage and hence its volatility. However, since it also reduces the risk premium elasticity, the net effect on interest rate volatility becomes relatively small. This provides further evidence that μ is a poorly identified parameter, unless it oscillates in low-value regions (as Figure 3 suggests) or leverage data are used in estimation as well. In column five we set $\sigma = 0.53$ following BGG, a level that is somewhat higher than values used in other studies on developed economies. Here, again, the volatilities of leverage and interest rate fall. As before, this can be linked to the change in the steady-state level of leverage. At this new level of σ, the long run level of leverage is lower relative to our estimated value (Figure 4) and, in consequence, it weakens the propagation mechanism of the financial accelerator.

Finally, in column six we conduct a counterfactual experiment where we turn off the near unit root process in TFP. To do so we set the persistence of the stationary productivity shock to $\rho_A = 0.95$, a somewhat standard value in the literature on business cycles in developed economies. The most important result of this experiment is that while consumption volatility is indeed much lower relative to the benchmark estimated case, neither leverage nor interest rate dynamics are significantly affected by this change. The latter is due to the fact that ρ_A does not affect the steady state of the financial accelerator. It affects the dynamics of leverage and interest rates solely by a slight change in the dynamics of investment (and hence in Q_t and K_{t+1} as well).[40]

[38] Incidentally, this counterfactual also provides an answer to why dropping Argentina and Ecuador from our sample does not change the results qualitatively. Although removing these two countries from the sample would reduce the standard deviation of the interest rate below 2%, this volatility would still be severalfold larger than the number generated under $\phi = 0.98$ and low steady state leverage.

[39] We also experimented with estimations in which ϕ would be calibrated to 0.98 and the remaining five parameters were estimated. The results were similar to this counterfactual in the sense that the leverage mechanism would virtually disappear and the interest rate volatility could not be matched. This alone would decrease the fit of the model significantly.

[40] With less persistent TFP shocks, *ceteribus paribus*, investment response is lower relative to the benchmark. In consequence, capital price and quantity react less relative to the net worth which increases somewhat the countercyclicality of leverage and

The result also signals that the near unit root persistence of TFP in the benchmark estimation stems from the need to match the high volatility of consumption as in Aguiar and Gopinath (2007a). It is important to stress that our GMM estimation points to this significant role of trend shocks despite the fact that the model already has a built-in microfounded financial accelerator mechanism. However, it has been argued by Aguiar and Gopinath (2007a) that trend shocks should be interpreted precisely as an emanation of deeper frictions. One could therefore conjecture that any frictions that can be recovered through nonstationary productivity processes are orthogonal to the financial frictions that we have studied here.

7 Concluding Remarks

The key task reported in this paper was to show how structural financial frictions in the form of a financial accelerator can provide a rationale for for some of the distinctive business cycle patterns in emerging markets. To this end, we embed a financial contract á la Bernanke et al. (1999) into a standard business cycle model of a small open economy. We show that many of the characteristics of cyclical fluctuations in emerging economies can be accounted for without the use of ad hoc or reduced-form processes for country interest rates. In particular, we apply the model to the data for emerging market economies and show that it reproduces the key stylized facts, notably the volatility and the countercyclicality of the risky country interest rate, without resorting to exogenous risk premium or foreign interest rate shocks. This is possible because of the countercyclical nature of the accelerator of a leveraged economy. In good times, i.e., after a positive productivity shock, net worth of firms goes up, which reduces the leverage as well as the fraction of bankrupt firms and hence drives the risk premium down. To rationalize this mechanism, we provide novel empirical evidence that assets-to-equity ratios of non-financial firms in emerging economies indeed tend to be countercyclical.

Our modeling technique also addresses another important point made by Oviedo (2005) and Aguiar and Gopinath (2007b), namely that fluctuations of interest rates should be linked to changes in productivity. For the same reason we abstain from incorporating a working capital requirement into our model, because, as shown by Chang and Fernández (2010), such friction is empirically not relevant relative to another friction that links the country interest rate to productivity. Nevertheless, the financial accelerator still shares, in a more structural form, part of the idea of the working capital constraint in that production (and therefore implicitly payments for input factors) is financed with borrowed funds.

The ongoing research program in financial frictions literature, including, e.g., papers of Christiano et al. (2010) and Fuentes-Albero (2012), provides evidence that the financial accelerator plays a statistically

interest rates.

significant role in explaining fluctuations in developed economies, e.g., in the U.S. and the Euro Area. Our work suggests that the mechanism may have an even higher potential in the context of business cycles in emerging economies.

Several extensions to our work deserve attention. First, because of the relatively simple analytical framework that we choose for tractability, we are disregarding other potentially important sources of financial frictions and/or propagation mechanisms. Modeling a full-fledged financial system could redirect attention to the net worth of financial intermediaries and their role in supplying loans to firms. This could also allow for a nontrivial role for the monetary authority which would act as an additional source of funding for the financial system, apart from deposits and foreign lending. That richer environment would also allow to introduce a wider array of sources of uncertainty. Second, while this work has shifted attention on the drivers of interest rates from the public to the private sector, a more comprehensive model should incorporate both elements so as to account for the large correlation between corporate and public interest rates documented in Section 2. Third, the study of corporate leverage certainly deserves more attention, both from an empirical and a theoretical perspective. A starting point of that analysis would probably be the fact that a large fraction of firms in emerging economies do not have access to a formal financial system but need to resort to informal sources of credit.

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Appendices

A Data

A.1 Emerging Economies, National Accounts and Interest Rates

The data for emerging economies come from Fernández and Zamora (2011). They build a wide panel dataset with information on National Accounts components and EMBI/CEMBI spreads. CEMBI index, which is the Emerging Market Corporate Bond Industrial Spread Over Benchmark Index, contains information on corporations, and it tracks only the liquid, tradable portion of the Emerging Markets USD corporate bond market.

The sources of National Accounts data are the International Monetary Fund's International Financial Statistics and The Economic Commission for Latin America and the Caribbean (ECLAC). ECLAC data are used only for recovering National Accounts for Ecuador after 3Q 1993. The source of EMBI and CEMBI is Bloomberg.

In all tables and calibrations in this paper, we use output series net of government expenditure, in order to make it consistent with our model. Since EMBI/CEMBI data are reported at different and higher than quarterly frequencies, we took appropriate simple averages. All variables except EMBI/CEMBI were seasonally adjusted using the the X-12 filter in Eviews and log-detrended using the HP filter.

National accounts data were computed by deflating the nominal series using GDP deflator-based inflation or, when not available, CPI inflation.

Real interest rates were obtained by substracting expected U.S. CPI inflation from the nominal rate. Expected inflation rate was computed as the average inflation in the current and three preceding months.

A.2 Developed Countries, National Accounts and Interest Rates

For developed economies we extend the Neumeyer and Perri (2005) dataset by both the number of countries and the period covered. Our extended set matches that of Aguiar and Gopinath (2007a). We also extend coverage until 3Q 2010. The data for the national accounts come from the Organization for Economic Cooperation and Development (OECD), quarterly national accounts section. We tested the data for any significant seasonal component. Whenever necessary, the data was then deseasonalized using the U.S. Census Bureau X-12 quarterly multiplicative seasonal adjustment method.[41] The OECD reports this series either with a base year or with chained prices. For Belgium, Denmark, Finland, Netherlands, Portugal, Spain and

[41] Seasonal adjustments were made for the national accounts of Belgium, Finland, Portugal and Sweden.

Sweden, we spliced the base year and chained series since neither was available at full length.

Consumption is households' final consumption. Whenever household consumption was not available,[42] we used private final consumption, which additionally includes final consumption of non-profit institutions. Investment is gross fixed capital formation. Government spending is government final consumption. Exports are goods and services exports. Imports are goods and services imports. Net exports is constructed as the difference between exports and imports. Gross Domestic Product (GDP) is constructed as the sum of consumption, investment and net exports, so we exclude government spending from GDP as in Fernández and Zamora (2011).

Following Neumeyer and Perri (2005) we computed the interest rates using data on 90-day corporate commercial papers, call money rates, or interbank lending rates. The data comes from the OECD database, Main Economic Indicators section (except for Switzerland where the source is Global Financial Data). For Australia and New Zealand the interest rate is the 90-day bank bill. For Austria we used the 90-day VIBOR. For Belgium, Denmark, Portugal and Spain it is the 90-day interbank rate. For Canada the nominal interest rate series we used is the 90-day corporate commercial paper. For Finland the interest rate is the 90-day HELIBOR. For Netherlands, from 3Q 1983 to 4Q 1985, we used the nominal interest rate reported by Neumeyer and Perri (2005) and spliced it with the 90-day AIBOR from 1Q 1986 to 4Q 2010. For Norway we used the 90-day NIBOR. The nominal interest rate for Sweden is the rate on 90-day treasuries and for Switzerland it is the 3-month LIBOR.

To calculate the real national accounts we deflated the series using the GDP Deflator (Australia, Austria, Canada and Norway) or the Consumer Price Index (Belgium, Denmark, Finland, Netherlands, New Zealand, Portugal, Spain, Sweden and Switzerland).

The real rate was obtained by subtracting the expected U.S. CPI inflation rate from the nominal interest rate. The expected inflation in period t was computed as the average of annual inflation in the current and the three preceding months. Finally, the real interest rate is reported in gross terms.

A.3 Leverage Dataset

The firm-level leverage dataset consists of all firms available in the Bloomberg database, for the 13 emerging economies considered, except for Ecuador, for which no data were found. We used the data for common stocks and preference stocks with the following trading status: active, delisted, expired, inactive, liquidated, suspended, unlisted, private companies, pending listing, pending symbol, postponed, acquired. We used the data for firms of all non-financial sectors, in particular: basic materials, consumer goods, consumer services, health care, industrials, oil and gas, technology, telecommunications. We only used firms for which the

[42] Household consumption was not available for Australia, Canada, Netherlands, Portugal and Switzerland.

primary exchange listing is the same as the country of domicile. The timing is quarterly, from 1Q 1995 until 3Q 2010. Figure 9 illustrates data availability, i.e., the number of firms used for a given country. In the figure, each country's observations are measured on both axes. As the number of observations for a country increases beyond 150, the axis switches from the right one to the left one. Series downloaded are: SHORT_AND_LONG_TERM_DEBT, CUR_MKT_CAP, as well as TOT_DEBT_TO_TOT_ASSETS. Leverage is then defined as

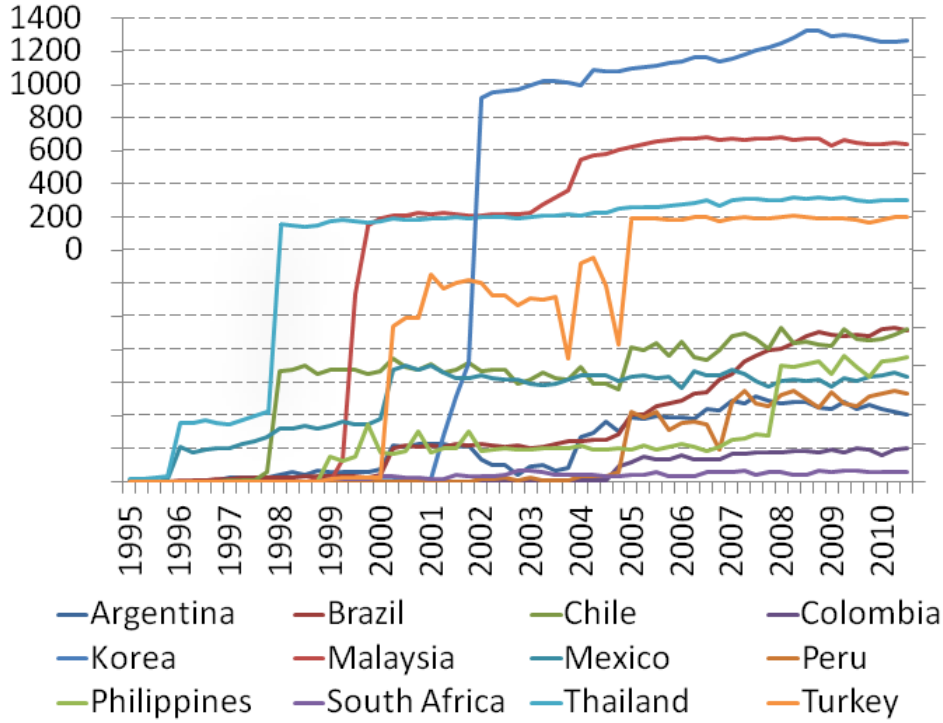

Figure 9: Number of firms used for the computation of leverage.

$$Lev_{i,t} = \frac{Equity_{i,t} + Debt_{i,t}}{Debt_{i,t}} \tag{A.1}$$

where i is the country and t is the quarter. Market leverage proxy is computed using market capitalization (CUR_MKT_CAP) as proxy for equity as well as (SHORT_AND_LONG_TERM_DEBT) as book value of debt. Aggregate leverage is then computed as a weighted average, where the weights are the relative firm sizes. In particular, in each period we sum market capitalizations across all firms of a given country. We do the same with debt. The data are filtered according to the following criteria: A) For market leverage, we take only data where both market capitalization and debt are available for a particular firm in a particular period. B) In statistical filtering, we first apply the following transformation of the leverage (computed on crude data): $\log(x-1)$. This transformed series resembles normal distribution. We then compute the mean and standard deviation for each country (all periods jointly) and discard observations which lie beyond the

95 percent confidence interval. C) In cases where 1 or at most 2 observations for the leverage were missing within the series, we performed linear interpolation. If the gaps were larger, we truncated the leverage series. D) The final filtered leverage series were HP filtered with smoothing parameter 1600.

A.4 Dividend Dataset

The data for dividends come from Bloomberg. We use the dividend payout ratio (DVD_PAYOUT_RATIO). The data extracted are annual (1995-2010), which has no bearing on our analysis given that we work with ratios. As with leverage, no data for Ecuador were available. The dividend payout ratio (DPR) is defined as the ratio of dividends paid in a given period over after-tax profit. This corresponds to the value

$$DPR_t = \frac{C_t^e}{V_t} = \frac{(1-\phi)\,V_t}{V_t} = 1-\phi$$

where the W_t^e term has been suppressed due to its very small value. We extract firm-level data from Bloomberg following the same criteria as with leverage (points 1.-4.). For each country we obtain an unbalanced panel of N firms and $T = 16$ years. For each country separately, we stack the data and drop observations which lie below the lowest 2.5 percentile and above the 97.5 percentile, hence working with the middle 95 percent of the empirical distributions. Country-wide dividend ratios reported in Table 10 of the paper are simple averages of the filtered observations. The total average is a simple average across 12 countries.

B Steady State

First, normalize $H_t^e \equiv 1\ \forall t$ (labor supply of entrepreneurs). Secondly, set $A = 1$ (technology), $H = 0.33$ (labor supply) as well as $R^* = (1.0077)^{1/4}$ (foreign interest rate).

The optimal $\bar{\omega}$ is found by maximizing entrepreneurs' return subject to lenders' zero-profit condition:

$$\max_{\bar{\omega},K,\lambda} \left[1-\Gamma(\bar{\omega})\right] R^K QK - \lambda \left\{R^*(QK-N) - \left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] R^K QK\right\}$$

with $Q = 1$. The first order conditions are:

$$\frac{\partial}{\partial \bar{\omega}} : -\Gamma_{\bar{\omega}}(\bar{\omega})\, R^K QK + \lambda \left[\Gamma_{\bar{\omega}}(\bar{\omega}) - \mu G_{\bar{\omega}}(\bar{\omega})\right] R^K QK = 0$$

$$\frac{\partial}{\partial K} : \left[1-\Gamma(\bar{\omega})\right] R^K Q - \lambda \left\{R^* Q - \left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] R^K Q\right\} = 0$$

$$\frac{\partial}{\partial \lambda} : R^*(QK-N) - \left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] R^K QK = 0$$

The solution yields:

$$\lambda\left(\bar{\omega}\right)=\frac{\Gamma_{\bar{\omega}}\left(\bar{\omega}\right)}{\Gamma_{\bar{\omega}}\left(\bar{\omega}\right)-\mu G_{\bar{\omega}}\left(\bar{\omega}\right)}$$

$$s\left(\bar{\omega}\right)\equiv\frac{\lambda\left(\bar{\omega}\right)}{1-\Gamma\left(\bar{\omega}\right)+\lambda\left[\Gamma\left(\bar{\omega}\right)-\mu G\left(\bar{\omega}\right)\right]}=\frac{R^{K}}{R^{*}}$$

and

$$k\left(\bar{\omega}\right)\equiv\frac{1-\Gamma\left(\bar{\omega}\right)+\lambda\left[\Gamma\left(\bar{\omega}\right)-\mu G\left(\bar{\omega}\right)\right]}{1-\Gamma\left(\bar{\omega}\right)}=\frac{QK}{N}$$

The second-order conditions which guarantee a maximum are given by the following inequality condition for the bordered hessian:

$$\det H=\begin{vmatrix} 0 & -\left(\Gamma_{\bar{\omega}}-\mu G_{\bar{\omega}}\right)R^{K}K & R^{*}-\left(\Gamma-\mu G\right)R^{K} \\ -\left(\Gamma_{\bar{\omega}}-\mu G_{\bar{\omega}}\right)R^{K}K & -\Gamma_{\bar{\omega}^{2}}R^{K}K+\lambda\left(\Gamma_{\bar{\omega}^{2}}-\mu G_{\bar{\omega}^{2}}\right)R^{K}K & -\Gamma_{\bar{\omega}}R^{K}+\lambda\left(\Gamma_{\bar{\omega}}-\mu G_{\bar{\omega}}\right)R^{K} \\ R^{*}-\left(\Gamma-\mu G\right)R^{K} & -\Gamma_{\bar{\omega}}R^{K}+\lambda\left(\Gamma_{\bar{\omega}}-\mu G_{\bar{\omega}}\right)R^{K} & 0 \end{vmatrix}>0$$

At this point, we have all the necessary parameter values to solve for optimal cutoff $\bar{\omega}$. We follow Gertler et al. (2007), correcting for the fact that we have a deterministic trend and borrowing directly from abroad at R^{*}. Specifically, take NSSS versions of equations 3.3, 3.19 combined with 3.8 as well as 3.7 combined with the zero-profit condition of lenders to get:

$$s\left(\bar{\omega}\right)-\frac{1-\delta}{R^{*}}=\frac{\alpha}{\Omega\left(1-\alpha\right)}\left[\frac{g}{R^{*}}\frac{1}{k\left(\bar{\omega}\right)}-\phi\left(1-\Gamma\left(\bar{\omega}\right)\right)s\left(\bar{\omega}\right)\right] \tag{B.1}$$

Now, we obtain

$$R^{K}=s\left(\bar{\omega}\right)R^{*} \tag{B.2}$$

Next, find the values for output Y and capital K by combining 3.17 with 3.3 to get

$$Y=\left\{\left[\frac{\alpha}{R^{K}-\left(1-\delta\right)}\right]^{\alpha}L^{\left(1-\alpha\right)}\right\}^{\frac{1}{1-\alpha}} \tag{B.3}$$

and

$$K=Y\frac{\alpha}{R^{K}-\left(1-\delta\right)} \tag{B.4}$$

Investment follows automatically from the assumption in 3.10:

$$I=\left(g-1+\delta\right)K \tag{B.5}$$

By definition, net worth is $N=K/k\left(\bar{\omega}\right)$.

Now, using 3.1, lending becomes simply

$$B=\left[k\left(\bar{\omega}\right)-1\right]N \tag{B.6}$$

Wages of entrepreneurs W^e and households W follow now from 3.19 and 3.20 respectively:

$$W^e = (1-\alpha)\,\Omega Y \tag{B.7}$$

$$W = (1-\alpha)\,(1-\Omega)\,\frac{Y}{H} \tag{B.8}$$

The value of the firm may be computed in at least two ways, e.g., using 3.8:

$$V = \frac{Ng - W^e}{\phi} \tag{B.9}$$

or, equivalently, from 3.7.

Entrepreneurs' consumption follows from 3.6:

$$C^e = (1-\phi)\,V \tag{B.10}$$

Risk premium comes from 3.15:

$$\Psi = \bar{\Psi} = \frac{g^{\sigma}}{\beta R^K} \tag{B.11}$$

Domestic consumption comes from $C = \frac{C}{Y} Y$.

Net exports NX comes from 3.21

$$NX = Y - C - C^e - I - \mu \int_0^{\bar{\omega}} \omega f(\omega)\,\mathrm{d}\omega\, R^K K \tag{B.12}$$

Foreign debt D now stems from 3.13

$$D = \frac{WH - C}{\Psi R^K - g} \tag{B.13}$$

Marginal utility of consumption comes from 3.16

Next, one can obtain the endogenous rescaling parameter τ in the GHH utility function, by combining the labor supply and labor demand equations, eliminating wages W and solving for τ:

$$\tau = \frac{(1-\alpha)\,(1-\Omega)\,Y}{H^{\gamma}}$$

Having τ, obtain the value for marginal utility of consumption λ as:

$$\lambda = \left(C - \tau\frac{H^{\gamma}}{\gamma}\right)^{-\sigma}$$

Finally, Table 13 summarizes all parameters which are found endogenously.

C Log-Linearized Model

Budget constraint

$$C\hat{C}_t - Yg\hat{D}_{t+1} = WH\left(\hat{W}_t + \hat{H}_t\right) - R^K D\bar{\Psi}\left(E_{t-1}\hat{R}^K_t + \hat{\Psi}_t\right) - \bar{\Psi}R^K Y\hat{D}_t \tag{C.1}$$

Table 13: Endogenously solved parameters

Variable or ratio	**Description**	**Comment**
$\bar{\Psi}$	constant in risk premium function	solved from 3.15
η	risk premium elasticity	solved from D.1
τ	parameter at GHH utility	solved from 3.12

where we define$\hat{D}_t = \frac{D_t - D}{Y}$ to account for a negative D in the non-stochastic steady state.

Risk premium

$$\hat{\Psi}_t = \frac{\tilde{\Psi} Y}{\bar{\Psi}} \hat{D}_t + \hat{\Phi}_t \tag{C.2}$$

Labor supply for GHH preferences

$$(\gamma - 1) \hat{H}_t - \hat{W}_t = 0 \tag{C.3}$$

Marginal utility of consumption for GHH preferences

$$\hat{\lambda}_t + \frac{\sigma C}{C - \tau \frac{H^\gamma}{\gamma}} \hat{C}_t - \frac{\sigma \tau H^\gamma}{C - \tau \frac{H^\gamma}{\gamma}} \hat{H}_t = 0 \tag{C.4}$$

Euler with foreign bonds

$$\hat{\lambda}_t = E_t \hat{R}^K_{t+1} + E_t \hat{\lambda}_{t+1} + \hat{\Psi}_{t+1} \tag{C.5}$$

Production function

$$\hat{Y}_t = \hat{A}_t + \alpha \hat{K}_t + (1 - \alpha) \hat{L}_t \tag{C.6}$$

Labor aggregation

$$\hat{L}_t = (1 - \Omega) \hat{H}_t \tag{C.7}$$

Entrepreneurial labor demand

$$\hat{Y}_t = \hat{W}^e_t \tag{C.8}$$

Labor demand

$$\hat{Y}_t - \hat{H}_t = \hat{W}_t \tag{C.9}$$

Investment funds

$$\hat{Q}_t + \hat{K}_{t+1} = \frac{N}{K} \hat{N}_{t+1} + \frac{B}{K} \hat{B}_{t+1} \tag{C.10}$$

Return on capital *ex post*

$$\hat{R}^K_t = \frac{\alpha \frac{Y}{K}}{R^K} \hat{Y}_t - \frac{\alpha \frac{Y}{K}}{R^K} \hat{K}_t + \frac{1 - \delta}{R^K} \hat{Q}_t - \hat{Q}_{t-1} \tag{C.11}$$

Interest rates

$$\frac{\bar{\omega}\left[\Gamma_{\bar{\omega}}-\mu G_{\bar{\omega}}\right]}{\Gamma-\mu G}\hat{\bar{\omega}}_t=\hat{R}_t^*-\hat{R}_t^K+\hat{B}_t-\hat{K}_t-\hat{Q}_{t-1} \tag{C.12}$$

where $\hat{\bar{\omega}}_t=\frac{\bar{\omega}_t-\bar{\omega}}{\bar{\omega}}$

Evolution of net worth

$$\hat{N}_{t+1}=\frac{\phi V}{Ng}\left(\hat{\upsilon}_t+\hat{V}_t\right)+\frac{W^e}{Ng}\hat{W}_t^e \tag{C.13}$$

Value of firms

$$V\hat{V}_t=\Xi R^K K\hat{R}_t^K+K\left(\Xi R^K-R^*\right)\left(\hat{Q}_{t-1}+\hat{K}_t\right)-\bar{\omega}\mu G_{\bar{\omega}}R^K K\hat{\bar{\omega}}_t-R^*\left(K-N\right)\hat{R}_t^*+R^*N\hat{N}_t \tag{C.14}$$

Entrepreneurial consumption

$$\hat{C}_t^e=\hat{V}_t \tag{C.15}$$

Motion of capital

$$g\hat{K}_{t+1}=\left(1-\delta\right)\hat{K}_t+\left(g-1+\delta\right)\hat{I}_t \tag{C.16}$$

Market clearing

$$\widehat{TB}_t=\left(1-TB\right)\hat{Y}_t-\frac{C}{Y}\hat{C}_t-\frac{C^e}{Y}\hat{C}_t^e-\frac{I}{Y}\hat{I}_t-\frac{\mu GR^K K}{Y}\left(\hat{R}_t^K+\hat{Q}_{t-1}+\hat{K}_t\right)-\frac{\bar{\omega}\mu G_{\bar{\omega}}R^K K}{Y}\hat{\bar{\omega}}_t \tag{C.17}$$

where

$$\widehat{TB}_t\equiv\frac{NX_t}{Y_t}-\frac{NX}{Y}=TB_t-TB$$

Home technology shock

$$\hat{A}_t=\rho_A\hat{A}_{t-1}+\epsilon_{A,t} \tag{C.18}$$

Optimal omega

$$\bar{\omega}\frac{\Gamma_{\bar{\omega}^2}\Lambda-\Sigma\Gamma_{\bar{\omega}}}{\Lambda^2}\left(\frac{R^K}{R^*}\Theta-1\right)E_t\hat{\bar{\omega}}_{t+1}+\frac{R^K}{R^*}\left[\left(1-\Gamma\right)+\frac{\Theta\Gamma_{\bar{\omega}}}{\Lambda}\right]\left(E_t\hat{R}_{t+1}^K-\hat{R}_{t+1}^*\right)=0 \tag{C.19}$$

where $\Xi=(1-\mu G)$, $\Theta=\Gamma-\mu G$, $\Lambda=\Gamma_{\bar{\omega}}-\mu G_{\bar{\omega}}$, $\Sigma=\Gamma_{\bar{\omega}^2}-\mu G_{\bar{\omega}^2}$ and $\Gamma\equiv\Gamma\left(\bar{\omega}\right)$, $\Gamma_{\bar{\omega}}\equiv\Gamma_{\bar{\omega}}\left(\bar{\omega}\right)=\frac{\partial}{\partial\bar{\omega}}\Gamma\left(\bar{\omega}\right)$, $\Gamma_{\bar{\omega}^2}\equiv\Gamma_{\bar{\omega}^2}\left(\bar{\omega}\right)=\frac{\partial^2}{\partial\bar{\omega}^2}\Gamma\left(\bar{\omega}\right)$ as well as $G\equiv G\left(\bar{\omega}\right)$, $G_{\bar{\omega}}\equiv G_{\bar{\omega}}\left(\bar{\omega}\right)=\frac{\partial}{\partial\bar{\omega}}G\left(\bar{\omega}\right)$, $G_{\bar{\omega}^2}\equiv G_{\bar{\omega}^2}\left(\bar{\omega}\right)=\frac{\partial^2}{\partial\bar{\omega}^2}G\left(\bar{\omega}\right)$.

Price of capital

$$\hat{Q}_t=\varphi g\hat{K}_{t+1}-\varphi g\hat{K}_t+\beta E_t\left\{\left(1-\delta\right)\hat{Q}_{t+1}-\varphi g^2\hat{K}_{t+2}+\varphi g^2\hat{K}_{t+1}\right\} \tag{C.20}$$

D Risk Premium Elasticity

The following computation derives the elasticity of the risk premium with respect to the leverage ratio, denoted as $\eta_{s,k}$. It closely follows Gertler et al. (2007). By definition, $s\left(\bar{\omega}\right)=\frac{\lambda(\bar{\omega})}{\Psi(\bar{\omega})}$ and $k\left(\bar{\omega}\right)=\frac{\Psi(\bar{\omega})}{1-\Gamma(\bar{\omega})}$, where $s(\bar{\omega}) = \frac{R^K}{R}$ is the risk premium and $k(\bar{\omega}) = \frac{QK}{N}$ is the leverage ratio. The elasticity is computed as $\eta_{s,k} = \frac{\mathrm{d}\log s}{\mathrm{d}\log k} = \frac{\mathrm{d}\log s}{\mathrm{d}\bar{\omega}} \frac{\mathrm{d}\bar{\omega}}{\mathrm{d}\log k}$.

$$\frac{\mathrm{d}\log s}{\mathrm{d}\bar{\omega}} = \frac{\mathrm{d}\left[\log \lambda(\bar{\omega}) - \log \Psi(\bar{\omega})\right]}{\mathrm{d}\bar{\omega}} = \frac{\lambda_{\bar{\omega}}(\bar{\omega})}{\lambda(\bar{\omega})} - \frac{\Psi_{\bar{\omega}}(\bar{\omega})}{\Psi(\bar{\omega})}$$

and

$$\frac{\mathrm{d}\log k}{\mathrm{d}\bar{\omega}} = \frac{\mathrm{d}\left[\log \Psi(\bar{\omega}) - \log(1 - \Gamma(\bar{\omega}))\right]}{\mathrm{d}\bar{\omega}} = \frac{\Psi_{\bar{\omega}}(\bar{\omega})}{\Psi(\bar{\omega})} + \frac{\Gamma_{\bar{\omega}}(\bar{\omega})}{1 - \Gamma(\bar{\omega})}$$

Combining, we obtain

$$\eta_{s,k} = \frac{\left[\frac{\lambda_{\bar{\omega}}(\bar{\omega})}{\lambda(\bar{\omega})} - \frac{\Psi_{\bar{\omega}}(\bar{\omega})}{\Psi(\bar{\omega})}\right]}{\left[\frac{\Psi_{\bar{\omega}}(\bar{\omega})}{\Psi(\bar{\omega})} + \frac{\Gamma_{\bar{\omega}}(\bar{\omega})}{1-\Gamma(\bar{\omega})}\right]} \tag{D.1}$$

E Log-Normal Distribution and Related Functions

We follow the standard notation established in the original BGG paper. The idiosyncratic productivity of a firm is denoted by ω. It is distributed log-normally, i.e., $\ln \omega \sim N\left(\mu_\omega, \sigma^2\right)$. Let $f(\omega)$ be the probability distribution function (pdf) of ω. It is given by

$$f(\omega) = \frac{1}{\omega\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln \omega - \mu_\omega)^2}{2\sigma^2}\right]$$

The cumulative distribution function (cdf) is

$$F(\bar{\omega}) = \int_0^{\bar{\omega}} f(\omega)\,\mathrm{d}\omega$$

In the model $E\omega$ is normalized to 1, therefore $\mu_\omega = -\frac{\sigma^2}{2}$.

The gross share of entrepreneurs' revenue that goes to the lenders is defined as

$$\Gamma(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega f(\omega)\,\mathrm{d}\omega + \bar{\omega}\int_{\bar{\omega}}^{\infty} f(\omega)\,\mathrm{d}\omega$$

and

$$G(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega f(\omega)\,\mathrm{d}\omega$$

$$\frac{\partial}{\partial\bar{\omega}}\Gamma(\bar{\omega}) = \Gamma_{\bar{\omega}}(\bar{\omega}) = 1 - F(\bar{\omega})$$

$$\frac{\partial}{\partial\bar{\omega}}G(\bar{\omega}) = G_{\bar{\omega}}(\bar{\omega}) = \bar{\omega} f(\bar{\omega})$$

$$\frac{\partial^2}{\partial\bar{\omega}^2}\Gamma(\bar{\omega}) = \Gamma_{\bar{\omega}^2}(\bar{\omega}) = -F'(\bar{\omega}) = -f(\bar{\omega})$$

$$\frac{\partial^2}{\partial\bar{\omega}^2}G(\bar{\omega}) = G_{\bar{\omega}^2}(\bar{\omega}) = f(\bar{\omega}) + \bar{\omega} f_{\bar{\omega}}(\bar{\omega})$$

where

$$f_{\bar{\omega}}(\bar{\omega}) = -\frac{1}{\bar{\omega}^2\sigma\sqrt{2\pi}}\left(1 + \frac{\ln\bar{\omega} + \frac{\sigma^2}{2}}{\sigma^2}\right)\exp\left\{-\frac{\left(\ln\bar{\omega} + \frac{\sigma^2}{2}\right)^2}{2\sigma^2}\right\}$$